Nucleon Compton scattering from the Dyson-Schwinger perspective

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Compton scattering off Protons and Light Nuclei: pinning down the nucleon polarizabilities

ECT*, Trento
August 1, 2013

Introduction

Goal: compute **nucleon's Compton scattering amplitude**
(and other things) from **quark-gluon substructure in QCD.**

- **Handbag** vs. nucleon (s- and u-channel) and meson (t-channel) **resonances?**
- **Electromagnetic gauge invariance** at the quark-gluon level?
- **Quark core** vs. **pion cloud?**
- **Tensor decomposition** for (on- and offshell) fermion two-photon vertex?

QCD's Green functions $\leftrightarrow$ "Dyson-Schwinger approach":
Nonperturbative, covariant, low and high energies, light and heavy quarks. But: **truncations!**

- **Baryon spectroscopy** from three-body Faddeev equation
  GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)
- **Elastic & transition form factors** for N and Δ
  GE, PRD 84 (2011); GE, Fischer, EPJ A48 (2012); GE, Nicmorus, PRD 85 (2012); ...
- **Tetraquark** interpretation for σ meson
  Heupel, GE, Fischer, PLB 718 (2012)
- **Compton scattering**
  GE, Fischer, PRD 85 (2012) & PRD 87 (2013)

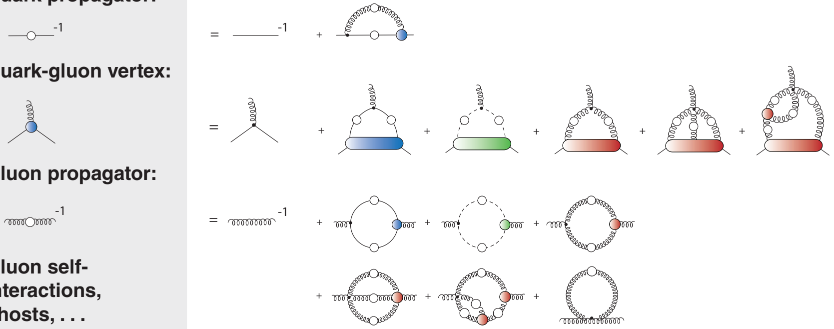
Dyson-Schwinger equations

QCD Lagrangian:
quarks, gluons (+ ghosts)

$$\mathcal{L} = \bar{\psi}(x)\,(i\partial\!\!\!/ + g A\!\!\!/ - M)\,\psi(x) - \frac{1}{4}\,F^a_{\mu\nu}\,F^{\mu\nu}_a$$

QCD & hadron properties are encoded in **QCD's Green functions.**
Their quantum equations of motion are the **Dyson-Schwinger equations (DSEs):**

- **Quark propagator:**
- **Quark-gluon vertex:**
- **Gluon propagator:**
- **Gluon self-interactions, ghosts, . . .**



Dyson-Schwinger equations

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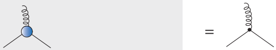
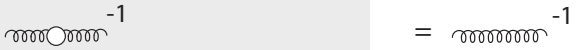


- **Gluon propagator:**



- **Gluon self-interactions, ghosts, . . .**

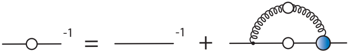
- **Truncation** ⇒ closed system, solveable.
  Ansätze for Green functions that are **not** solved (based on pQCD, lattice, FRG, ...)

- **Applications:**
  Origin of confinement,
  QCD phase diagram,
  Hadron physics

Dynamical quark mass

- **Dynamical chiral symmetry breaking:** generates "constituent-quark masses"

- Realized in **quark Dyson-Schwinger eq:**

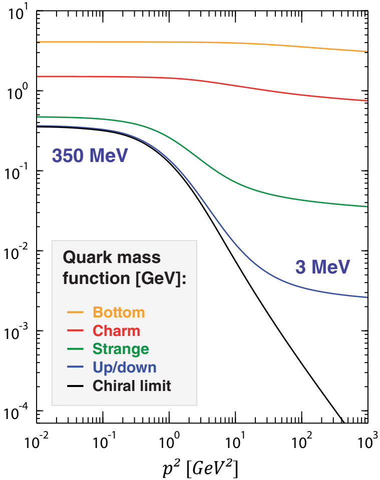


If (gluon propagator $\times$ quark-gluon vertex) is strong enough ($\alpha > \alpha_{\text{crit}}$): momentum-dependent quark mass $M(p^2)$

- Already visible in simpler models (NJL, Munczek-Nemirovsky)

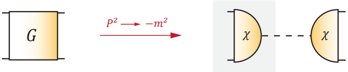
- Mass generation for **light hadrons**

Fischer, J. Phys. G 32 (2006)



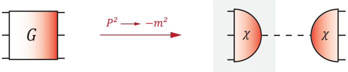
Hadrons: poles in Green functions

- **Quark four-point function:**
  $\langle 0|\mathrm{T}\,\psi(x_1)\,\bar{\psi}(x_2)\,\psi(x_3)\,\bar{\psi}(x_4)|0\rangle$



Bethe-Salpeter WF:
$\langle 0|\,T\,\psi(x_1)\,\bar{\psi}(x_2)\,|H\rangle$

- **Quark six-point function:**



Faddeev WF

Hadrons: poles in Green functions

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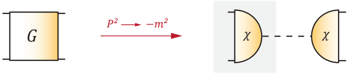

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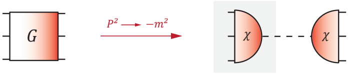
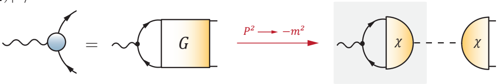
- **Quark six-point function:**



Faddeev WF

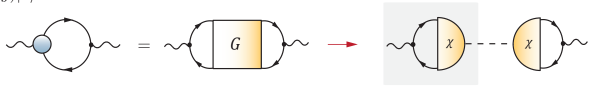
- **Quark-antiquark vertices:** (Currents: $J^\mu = \bar\psi\,\Gamma^\mu\psi$)
  $\langle 0|\mathrm{T}\,J^\mu(x)\,\psi(x_1)\,\bar\psi(x_2)|0\rangle$



Decay constant:
$\langle 0|\,J^\mu\,|H\rangle$

Quark-photon vertex has ρ-meson poles: 'vector-meson dominance'

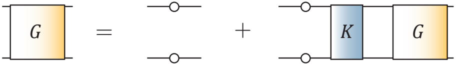
- **Current correlators:**
  $\langle 0|\mathrm{T}\,J^\mu(x)\,J^\nu(y)|0\rangle$



(→ Lattice QCD)

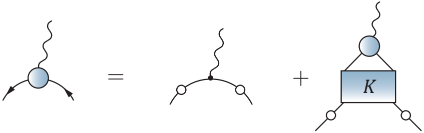
Bethe-Salpeter equations

- Inhomogeneous BSE
  for **quark four-point function:**



- Homogeneous BSE
  for **bound-state wave function:**

- Inhomogeneous BSE
  for **quark-antiquark vertices:**



Analogy: geometric series

$$f(x) = 1 + x f(x) \quad \Rightarrow \quad f(x) = \tfrac{1}{1-x}$$

$$|x| < 1 \quad \Rightarrow \quad f(x) = 1 + x + x^2 + \dots$$

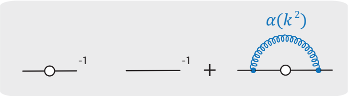
What's the kernel K?

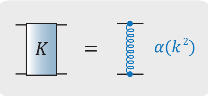
Related to Green functions
via **symmetries:** CVC, PCAC
$\Rightarrow$ vector, axialvector WTIs

Relate **K** with quark propagator
and quark-gluon vertex

Structure of the kernel

Rainbow-ladder: tree-level vertex + effective coupling





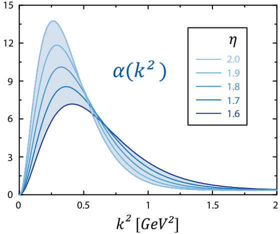
√ **DCSB, CVC, PCAC**

⇒ mass generation

⇒ Goldstone theorem, massless pion in χL

⇒ em. current conservation

⇒ Goldberger-Treiman



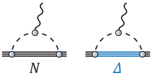
Ansatz for effective coupling:

Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

$$\alpha(k^2) = \alpha_{\mathrm{IR}}\left(k^2/\Lambda^2, \eta\right) + \alpha_{\mathrm{UV}}(k^2)$$

Adjust infrared scale Λ to physical observable, keep width η as parameter

No pion cloud, no flavor dependence, no $U_A(1)$ anomaly, no dynamical decay widths



, etc.

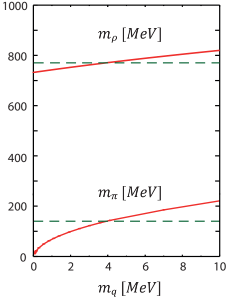
Pion cloud: need infinite summation of t-channel gluons

Mesons

- **Pseudoscalar & vector mesons:** rainbow-ladder is good. Masses, form factors, decays, $\pi\pi$ scattering lengths, PDFs

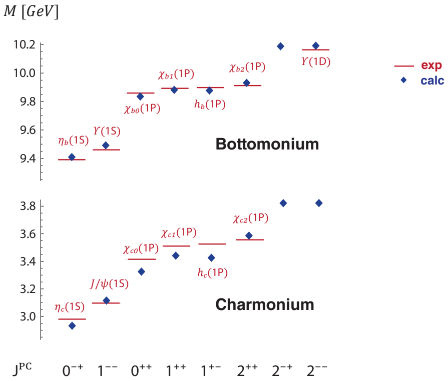
  Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999); Bashir et al., Commun. Theor. Phys. 58 (2012)

  Pion is Goldstone boson, satisfies GMOR: $m_\pi{}^2 \sim m_q$



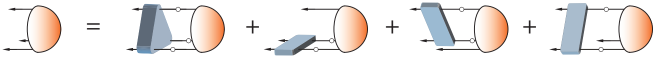
- Need to go **beyond rainbow-ladder** for excited, scalar, axialvector mesons, η-η', etc.

- **Heavy mesons** Blank, Krassnigg, PRD 84 (2011)



Baryons

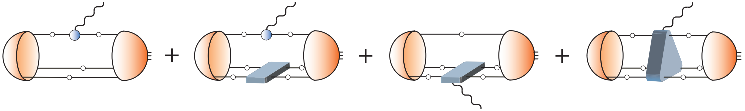
Covariant Faddeev equation: kernel contains 2PI and 3PI parts



Current matrix element: $\langle H|J^\mu|H\rangle = \bar{\chi}\,(G^{-1})^\mu\,\chi$

- Impulse approximation + gauged kernel $(G^{-1})^\mu = (G_0^{-1})^\mu - K^\mu$

'Gauging of equations':
Kvinikhidze, Blankleider, PRC 60 (1999)
Oettel, Pichowsky, von Smekal, EPJ A 8 (2000)



Truncation:

- **Quark-quark correlations** only (dominant structure in baryons?)
- Rainbow-ladder **gluon exchange**
- But **full Poincaré-covariant structure** of Faddeev amplitude retained

$\longrightarrow$ Same input as for mesons, quark from DSE, no additional parameters!

Baryons

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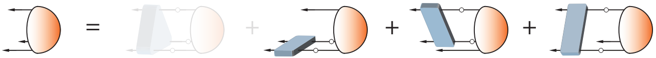

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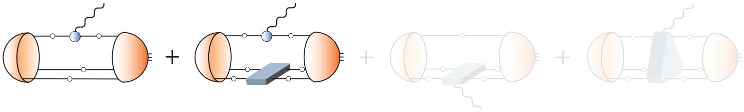

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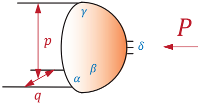
Faddeev wave function

s	l	T_{ij}
$1/2$	0	$\mathbb{1}\otimes\mathbb{1}$ **s waves (8)**
$1/2$	0	$\gamma^\mu_T\otimes\gamma^\mu_T$
$1/2$	1	$\mathbb{1}\otimes\frac{1}{2}[\slashed{p},\slashed{q}]$ **p waves (36)**
$1/2$	1	$\mathbb{1}\otimes\slashed{p}$
$1/2$	1	$\mathbb{1}\otimes\slashed{q}$
$1/2$	1	$\gamma^\mu_T\otimes\gamma^\mu_T\,\frac{1}{2}[\slashed{p},\slashed{q}]$
$1/2$	1	$\gamma^\mu_T\otimes\gamma^\mu_T\,\slashed{p}$
$1/2$	1	$\gamma^\mu_T\otimes\gamma^\mu_T\,\slashed{q}$
$3/2$	1	$3(\slashed{p}\otimes\slashed{q}-\slashed{q}\otimes\slashed{p})-\gamma^\mu_T\otimes\gamma^\mu_T\,[\slashed{p},\slashed{q}]$
$3/2$	1	$3\,\slashed{p}\otimes\mathbb{1}-\gamma^\mu_T\otimes\gamma^\mu_T\,\slashed{p}$
$3/2$	1	$3\,\slashed{q}\otimes\mathbb{1}-\gamma^\mu_T\otimes\gamma^\mu_T\,\slashed{q}$
$3/2$	2	$3\,\slashed{p}\otimes\slashed{p}-\gamma^\mu_T\otimes\gamma^\mu_T$ **d waves (20)**
$3/2$	2	$\slashed{p}\otimes\slashed{p}+2\,\slashed{q}\otimes\slashed{q}-\gamma^\mu_T\otimes\gamma^\mu_T$
$3/2$	2	$\slashed{p}\otimes\slashed{q}+\slashed{q}\otimes\slashed{p}$
$3/2$	2	$\slashed{q}\otimes[\slashed{q},\slashed{p}]-\frac{1}{2}\gamma^\mu_T\otimes[\gamma^\mu_T,\slashed{p}]$
$3/2$	2	$\slashed{p}\otimes[\slashed{p},\slashed{q}]-\frac{1}{2}\gamma^\mu_T\otimes[\gamma^\mu_T,\slashed{q}]$

$$\chi(x_1,x_2,x_3)=\langle 0|\,T\,\psi(x_1)\,\psi(x_2)\,\psi(x_3)\,|N\rangle$$

Momentum space:
Jacobi coordinates p, q, P
⇒ 5 Lorentz invariants
⇒ 64 Dirac basis elements



$$\chi(p,q,P)=\sum_k \boxed{f_k(p^2,\,q^2,\,p\cdot q,\,p\cdot P,\,q\cdot P)\quad \textbf{Momentum}}$$
$$\boxed{\tau^k_{\alpha\beta\gamma\delta}(p,q,P)\quad \textbf{Dirac}}\;\boxed{\otimes\ \textbf{Flavor}\ \otimes\ \textbf{Color}}$$

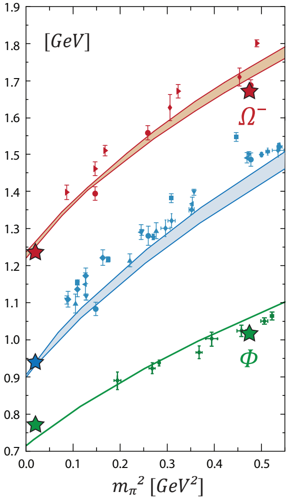
Complete, orthogonal **Dirac tensor basis**
(partial-wave decomposition in nucleon rest frame):
GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)

$$T_{ij}\,(\Lambda_\pm\gamma_5 C\otimes\Lambda_+)$$
$$(\gamma_5\otimes\gamma_5)\,T_{ij}\,(\Lambda_\pm\gamma_5 C\otimes\Lambda_+)$$

$$(A\otimes B)_{\alpha\beta\gamma\delta}=A_{\alpha\beta}\,B_{\gamma\delta}$$

Baryon masses

- Good agreement with experiment & lattice. Pion mass is also calculated.
- Same kernel as for mesons, scale set by f_π. Full covariant wave functions, no further parameters or approximations.
- Masses not sensitive to effective interaction.
- **Diquark clustering in baryons:** similar results in quark-diquark approach
  Oettel, Alkofer, von Smekal, EPJ A8 (2000)
  GE, Cloet, Alkofer, Krassnigg, Roberts, PRC 79 (2009)
- **Excited baryons** (e.g. Roper): also quark-diquark structure?



Delta mass:
Sanchis-Alepuz *et al.*, PRD 84 (2011)

Nucleon mass:
GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010);
GE, PRD 84 (2011)

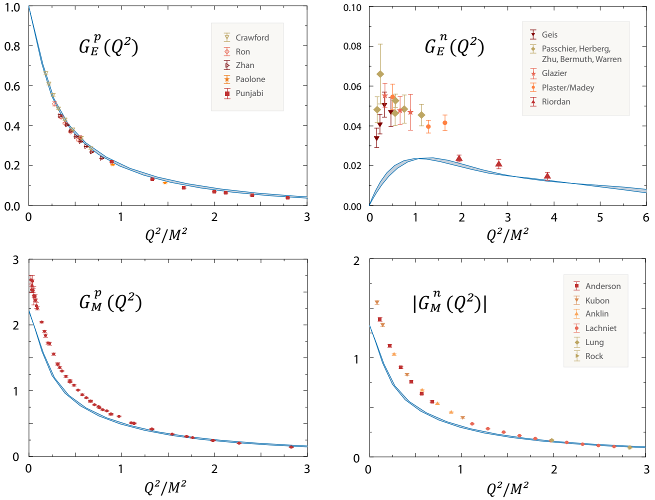
ρ–meson mass:
Maris & Tandy, PRC 60 (1999)

Electromagnetic form factors

Nucleon em. FFs
vs. momentum transfer
GE, PRD 84 (2011)

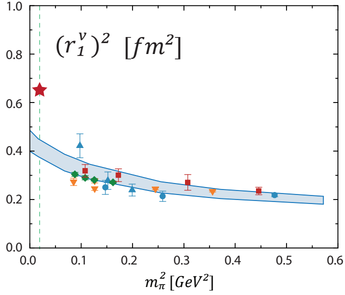
- Agreement with data at larger Q^2 and lattice at larger quark masses
- **Missing pion cloud** below 1–2 GeV^2, in chiral region

$\sim$ **nucleon quark core** without pion effects



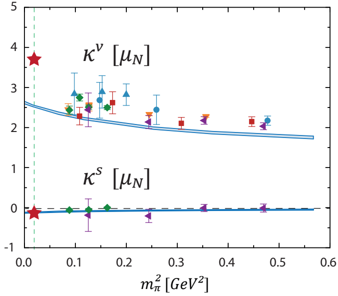
Electromagnetic form factors

Nucleon charge radii:
isovector (p-n) Dirac (F1) radius



- **Pion-cloud effects** missing in chiral region (⇒ divergence!), agreement with lattice at larger quark masses.

Nucleon magnetic moments:
isovector (p-n), isoscalar (p+n)



- **But:** pion-cloud **cancels** in κ^s ⟺ **quark core**

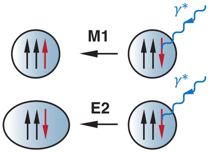
Exp: $\kappa^s = -0.12$
Calc: $\kappa^s = -0.12(1)$ **!!**

Nucleon-Δ-γ transition

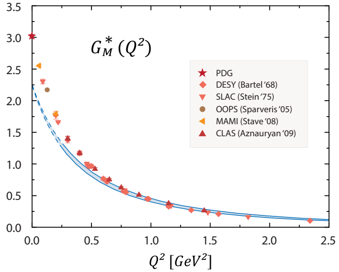
- **Magnetic dipole form factor** $G_M^\star$ dominant, quark spin flip. As expected: "Core + 25% pion cloud"

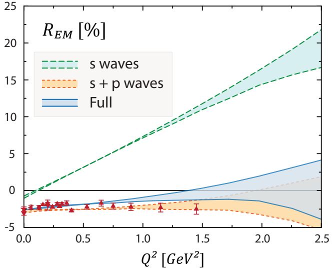
- **Electric quadrupole transition** R_{EM} small & negative, encodes deformation. Perturbative QCD: $R_{EM} \to 1$, Quark model: need **d waves** or **pion cloud.**

  But: subleading tensor structures important, **Quark OAM (p waves)** by Poincaré covariance!

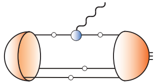
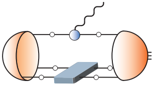


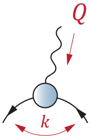
GE & Nicmorus, PRD 85 (2012)





Quark-photon vertex

Current matrix element: $\langle H | J^\mu | H \rangle =$  $+$ 



Vector WTI $Q^\mu \, \Gamma^\mu(k,Q) = S^{-1}(k_+) - S^{-1}(k_-)$
determines vertex up to transverse parts:

$$\Gamma^\mu(k,Q) = \Gamma^\mu_{\mathrm{BC}}(k,Q) + \Gamma^\mu_{\mathrm{T}}(k,Q)$$

- **Ball-Chiu vertex**, completely specified by dressed fermion propagator: Ball, Chiu, PRD 22 (1980)

$$\Gamma^\mu_{\mathrm{BC}}(k,Q) = i\gamma^\mu \, \Sigma_A + 2k^\mu (i \not{k} \, \Delta_A + \Delta_B)$$

$$\Sigma_A := \frac{A(k_+^2) + A(k_-^2)}{2}, \quad \Delta_A := \frac{A(k_+^2) - A(k_-^2)}{k_+^2 - k_-^2}, \quad \Delta_B := \frac{B(k_+^2) - B(k_-^2)}{k_+^2 - k_-^2}$$

- **Transverse part:** free of kinematic singularities, tensor structures $\sim Q, Q^2, Q^3$, contains meson poles
Kizilersu, Reenders, Pennington, PRD 92 (1995); GE, Fischer, PRD 87 (2013)

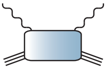
$$t^{\mu\nu}_{ab} := a \cdot b \, \delta^{\mu\nu} - b^\mu a^\nu$$

Dominant	$\tau_1^\mu = t^{\mu\nu}_{QQ} \, \gamma^\nu \,,$	$\tau_5^\mu = t^{\mu\nu}_{QQ} \, ik^\nu \,,$
	$\tau_2^\mu = t^{\mu\nu}_{QQ} \, k \cdot Q \, \frac{i}{2} [\gamma^\nu, \not{k}] \,,$	$\tau_6^\mu = t^{\mu\nu}_{QQ} \, k^\nu \not{k} \,,$
Anomalous magnetic moment	$\tau_3^\mu = \frac{i}{2} [\gamma^\mu, \not{Q}] \,,$	$\tau_7^\mu = t^{\mu\nu}_{Qk} \, k \cdot Q \, \gamma^\nu \,,$ Curtis, Pennington, PRD 42 (1990)
	$\tau_4^\mu = \frac{1}{6} [\gamma^\mu, \not{k}, \not{Q}] \,,$	$\tau_8^\mu = t^{\mu\nu}_{Qk} \, \frac{i}{2} [\gamma^\nu, \not{k}] \,.$

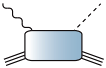
Hadron scattering

Can we extend this to **four-body scattering** processes?

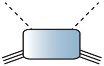
GE, Fischer, PRD 85 (2012)



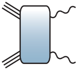
Compton scattering, DVCS, 2γ physics



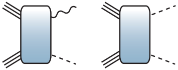
Meson photo- and electroproduction



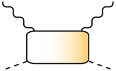
Nucleon-pion scattering



$\bar{p}p \to \gamma\gamma^*$
annihilation



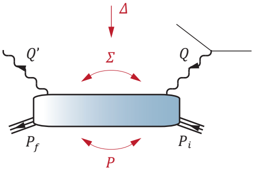
Meson production



Pion Compton scattering

$\Rightarrow$ Nonperturbative description of hadron-photon and hadron-meson scattering

Nucleon Compton scattering

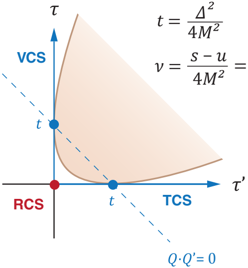


$$\tau = \frac{Q^2}{4M^2}$$

$$\tau' = \frac{Q'^2}{4M^2}$$

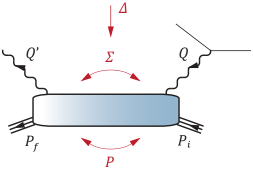
$$t = \frac{\Delta^2}{4M^2}$$

$$\nu = \frac{s-u}{4M^2} = -\frac{\Sigma\cdot P}{M^2}$$



- **RCS, VCS:**
  nucleon (generalized) polarizabilities
- **DVCS:**
  factorization & handbag dominance, GPDs
- **Forward limit:**
  structure functions in DIS
- **Timelike region:**
  $p\bar{p}$ annhihilation at PANDA@FAIR
- **Spacelike region:**
  two-photon corrections to nucleon form factors

Nucleon Compton scattering

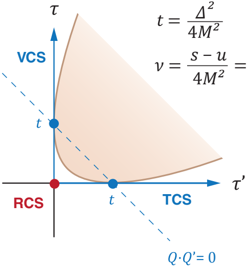


$$\tau = \frac{Q^2}{4M^2}$$

$$\tau' = \frac{Q'^2}{4M^2}$$

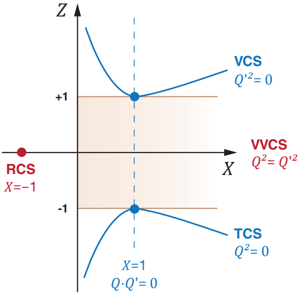
$$t = \frac{\Delta^2}{4M^2}$$

$$\nu = \frac{s-u}{4M^2} = -\frac{\Sigma\cdot P}{M^2}$$

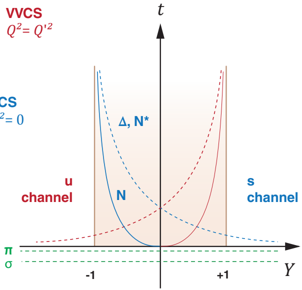


4-dim phase space simpler in Lorentz-invariant hyperspherical variables **t, X, Y, Z:**

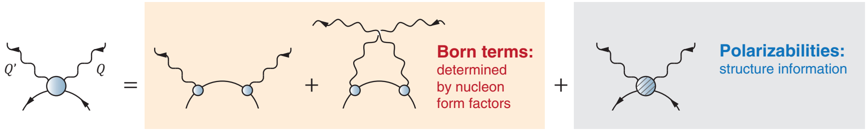
$$\frac{\Delta^2}{4M^2} = t\,, \quad \frac{\Sigma^2}{M^2} =: tX\,, \quad \widehat{P}\cdot\widehat{\Sigma_T} =: Y\,, \quad \widehat{\Sigma}\cdot\widehat{\Delta} =: Z$$



Mandelstam plane in RCS:



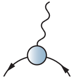
Compton scattering



- All direct measurements in kinematic limits (RCS, VCS, forward limit).
- Em. gauge invariance $\Rightarrow$ Compton amplitude is **fully transverse.** **Analyticity** constrains 1PI part in these limits (low-energy theorem).
- Polarizabilities = coefficients of tensor structures that vanish like $\sim Q^\mu Q'^\nu,\; Q^\mu Q^\nu,\; Q'^\mu Q'^\nu, \dots$
- Need tensor basis free of kinematic singularities (18 elements). Complicated...

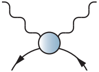
Bardeen, Tung, Phys. Rev. 173 (1968)
Perrottet, Lett. Nuovo Cim. 7 (1973)
Tarrach, Nuovo Cim. 28 A (1975)
Drechsel et al., PRC 57 (1998)
L'vov et al., PRC 64 (2001)
Gorchtein, PRC 81 (2010)
Belitsky, Mueller, Ji, 1212.6674 [hep-ph]
...

Tensor basis?



$$\tau_1 = \gamma^\mu \qquad \tau_2 = \sigma^{\mu\nu}\, Q^\nu$$

Transversality, analyticity and **Bose symmetry** makes the construction extremely difficult...



Tarrach, Nuovo Cim. 28 (1975)

$$\begin{aligned}
T_1 &= g_{\nu\mu}\,, & T_{18} &= (P_\nu k_\mu - P_\mu k'_\nu)\hat{K}\,,\\
T_2 &= k_\nu k'_\mu\,, & T_{19} &= (P_\nu k'_\mu + P_\mu k_\nu)\hat{K}\,,\\
T_3 &= k'_\nu k_\mu\,, & T_{20} &= (P_\nu k'_\mu - P_\mu k_\nu)\hat{K}\,,\\
T_4 &= k_\nu k_\mu + k'_\nu k'_\mu\,, & T_{21} &= P_\nu\gamma_\mu + P_\mu\gamma_\nu\,,\\
T_5 &= k_\nu k_\mu - k'_\nu k'_\mu\,, & T_{22} &= P_\nu\gamma_\mu - P_\mu\gamma_\nu\,,\\
T_6 &= P_\nu P_\mu\,, & T_{23} &= k_\nu\gamma_\mu + k'_\mu\gamma_\nu\,,\\
T_7 &= P_\nu k_\mu + P_\mu k'_\nu\,, & T_{24} &= k_\nu\gamma_\mu - k'_\mu\gamma_\nu\,,\\
T_8 &= P_\nu k_\mu - P_\mu k'_\nu\,, & T_{25} &= k'_\nu\gamma_\mu + k_\mu\gamma_\nu\,,\\
T_9 &= P_\nu k'_\mu + P_\mu k_\nu\,, & T_{26} &= k'_\nu\gamma_\mu - k_\mu\gamma_\nu\,,\\
T_{10} &= P_\nu k'_\mu - P_\mu k_\nu\,, & T_{27} &= (P_\nu\gamma_\mu + P_\mu\gamma_\nu)\hat{K} - \hat{K}(P_\nu\gamma_\mu + P_\mu\gamma_\nu)\,,\\
T_{11} &= g_{\nu\mu}\hat{K}\,, & T_{28} &= (P_\nu\gamma_\mu - P_\mu\gamma_\nu)\hat{K} - \hat{K}(P_\nu\gamma_\mu - P_\mu\gamma_\nu)\,,\\
T_{12} &= k_\nu k'_\mu\hat{K}\,, & T_{29} &= (k_\nu\gamma_\mu + k'_\mu\gamma_\nu)\hat{K} - \hat{K}(k_\nu\gamma_\mu + k'_\mu\gamma_\nu)\,,\\
T_{13} &= k'_\nu k_\mu\hat{K}\,, & T_{30} &= (k_\nu\gamma_\mu - k'_\mu\gamma_\nu)\hat{K} - \hat{K}(k_\nu\gamma_\mu - k'_\mu\gamma_\nu)\,,\\
T_{14} &= (k_\nu k_\mu + k'_\nu k'_\mu)\hat{K}\,, & T_{31} &= (k'_\nu\gamma_\mu + k_\mu\gamma_\nu)\hat{K} - \hat{K}(k'_\nu\gamma_\mu + k_\mu\gamma_\nu)\,,\\
T_{15} &= (k_\nu k_\mu - k'_\nu k'_\mu)\hat{K}\,, & T_{32} &= (k'_\nu\gamma_\mu - k_\mu\gamma_\nu)\hat{K} - \hat{K}(k'_\nu\gamma_\mu - k_\mu\gamma_\nu)\,,\\
T_{16} &= P_\nu P_\mu\hat{K}\,, & T_{33} &= \gamma_\nu\gamma_\mu - \gamma_\mu\gamma_\nu\,,\\
T_{17} &= (P_\nu k_\mu + P_\mu k'_\nu)\hat{K}\,, & T_{34} &= (\gamma_\nu\gamma_\mu - \gamma_\mu\gamma_\nu)\hat{K} + \hat{K}(\gamma_\nu\gamma_\mu - \gamma_\mu\gamma_\nu)\,,
\end{aligned}$$

$$\begin{aligned}
\tau_1 &= k\cdot k'\,T_1 - T_3\,,\\
\tau_2 &= k^2k'^2T_1 + k\cdot k'\,T_2 - \frac{k^2+k'^2}{2}T_4 + \frac{k^2-k'^2}{2}T_5\,,\\
\tau_3 &= (P\cdot K)^2T_1 + k\cdot k'\,T_6 - P\cdot K T_7\,,\\
\tau_4 &= P\cdot K(k^2+k'^2)T_1 - P\cdot KT_4 - \frac{k^2+k'^2}{2}T_7 + \frac{k^2-k'^2}{2}T_8 + k\cdot k'\,T_9\,,\\
\tau_5 &= -P\cdot K(k^2-k'^2)T_1 + P\cdot KT_5 + \frac{k^2-k'^2}{2}T_7 - \frac{k^2+k'^2}{2}T_8 + k\cdot k'\,T_{10}\,,\\
\tau_6 &= P\cdot KT_2 - \frac{k^2+k'^2}{4}T_9 - \frac{k^2-k'^2}{4}T_{10} - MT_{12} + M\frac{k^2+k'^2}{4}T_{23} -\\
&\quad - M\frac{k^2-k'^2}{4}T_{24} + \frac{k^2-k'^2}{8}T_{29} - \frac{k^2+k'^2}{8}T_{30} - \frac{k^2k'^2}{4}T_{33}\,,\\
\tau_7 &= 8T_{16} - 4P\cdot KT_{21} + P\cdot KT_{34}\,,\\
\tau_8 &= T_{19} + \frac{k^2-k'^2}{2}T_{22} - P\cdot KT_{23} + \frac{k^2+k'^2}{8}T_{34}\,,\\
\tau_9 &= T_{20} - \frac{k^2+k'^2}{2}T_{22} + P\cdot KT_{24} - \frac{k^2-k'^2}{8}T_{34}\,,\\
\tau_{10} &= -8k\cdot k'\,T_6 + 4P\cdot KT_7 + 4Mk\cdot k'\,T_{21} - 4MP\cdot KT_{25} -\\
&\quad - 2P\cdot KT_{32} - 2k\cdot k'\,P\cdot KT_{33} + Mk\cdot k'\,T_{34}\,,\\
\tau_{11} &= T_{18} - k\cdot k'\,T_{22} + P\cdot KT_{26}\,,\\
\tau_{12} &= P\cdot KT_4 - \frac{k^2-k'^2}{2}T_8 - k\cdot k'\,T_9 - MT_{14} + Mk\cdot k'\,T_{23} -\\
&\quad - M\frac{k^2-k'^2}{2}T_{26} - \frac{k^2+k'^2}{4}T_{32} - k\cdot k'\frac{k^2+k'^2}{4}T_{33}\,,\\
\tau_{13} &= P\cdot KT_5 - \frac{k^2+k'^2}{2}T_8 + k\cdot k'\,T_{10} - MT_{15} + Mk\cdot k'\,T_{24} -\\
&\quad - M\frac{k^2+k'^2}{2}T_{26} - \frac{k^2-k'^2}{4}T_{32} - k\cdot k'\frac{k^2-k'^2}{4}T_{33}\,,
\end{aligned}$$

$$\begin{aligned}
\tau_{14} &= 2P\cdot KT_8 - 2Mk\cdot k'\,T_{22} + 2MP\cdot KT_{26} - k\cdot k'\,T_{27} + P\cdot KT_{31}\,,\\
\tau_{15} &= -(k^2-k'^2)T_7 + (k^2+k'^2)T_8 - 2k\cdot k'\,T_{10} - 2Mk\cdot k'\,T_{24} +\\
&\quad + M(k^2-k'^2)T_{25} + M(k^2+k'^2)T_{26} - k\cdot k'\,T_{29} +\\
&\quad + \frac{k^2+k'^2}{2}T_{31} + \frac{k^2-k'^2}{2}T_{32}\,,\\
\tau_{16} &= -(k^2+k'^2)T_7 + (k^2-k'^2)T_8 + 2k\cdot k'\,T_9 - 2Mk\cdot k'\,T_{23} +\\
&\quad + M(k^2+k'^2)T_{25} + M(k^2-k'^2)T_{26} - k\cdot k'\,T_{30} +\\
&\quad + \frac{k^2-k'^2}{2}T_{31} + \frac{k^2+k'^2}{2}T_{32}\,,\\
\tau_{17} &= -4P\cdot KT_1 + 2T_7 + 4MT_{11} - 2MT_{25} + T_{32} + k\cdot k'\,T_{33}\,,\\
\tau_{18} &= 4T_{17} - 4P\cdot KT_{25} + k\cdot k'\,T_{34}\,.\\
\tau_{19} &= \frac{1}{k\cdot k'}[2(P\cdot K)^2\tau_2 + 2k^2k'^2\tau_3 - P\cdot K(k^2+k'^2)\tau_4 - P\cdot K(k^2-k'^2)\tau_5] =\\
&= 2(P\cdot K)^2T_2 + 2k^2k'^2T_6 - P\cdot K(k^2+k'^2)T_9 - P\cdot K(k^2-k'^2)T_{10}\,,\\
\tau_{20} &= \frac{1}{4k\cdot k'}[(k^2-k'^2)\tau_{10} - 2(k^2+k'^2)\tau_{14} + 4P\cdot K\tau_{15}] =\\
&= -2(k^2-k'^2)T_6 - 2P\cdot KT_{10} + M(k^2-k'^2)T_{21} + M(k^2+k'^2)T_{22} -\\
&\quad - 2MP\cdot KT_{24} + \frac{k^2+k'^2}{2}T_{27} - P\cdot KT_{29} -\\
&\quad - P\cdot K\frac{k^2-k'^2}{2}T_{33} + M\frac{k^2-k'^2}{4}T_{34}\,,\\
\tau_{21} &= \frac{1}{4k\cdot k'}[(k^2+k'^2)\tau_{10} - 2(k^2-k'^2)\tau_{14} + 4P\cdot K\tau_{16}] =\\
&= -2(k^2+k'^2)T_6 + 2P\cdot KT_9 + M(k^2+k'^2)T_{21} + M(k^2-k'^2)T_{22} -\\
&\quad - 2MP\cdot KT_{23} + \frac{k^2-k'^2}{2}T_{27} - P\cdot KT_{30} -\\
&\quad - P\cdot K\frac{k^2+k'^2}{2}T_{33} + M\frac{k^2+k'^2}{4}T_{34}\,.
\end{aligned}$$

Transverse tensor basis for $\Gamma^{\mu\nu}(p,Q,Q')$

- Generalize transverse projectors:
$$t^{\mu\nu}_{ab} := a\cdot b\,\delta^{\mu\nu} - b^\mu a^\nu \qquad a,b\in\{p,Q,Q'\}$$
$$\varepsilon^{\mu\nu}_{ab} := \gamma_5\,\varepsilon^{\mu\nu\alpha\beta} a^\alpha b^\beta \qquad \text{(exhausts all possibilities)}$$

- Apply Bose-(anti-)symmetric combinations
$$\mathsf{E}^{\mu\alpha,\beta\nu}_\pm(a,b) := \tfrac{1}{2}\left(\varepsilon^{\mu\alpha}_{Q'a'}\,\varepsilon^{\beta\nu}_{bQ} \pm \varepsilon^{\mu\alpha}_{Q'b'}\,\varepsilon^{\beta\nu}_{aQ}\right)$$
$$\mathsf{F}^{\mu\alpha,\beta\nu}_\pm(a,b) := \tfrac{1}{2}\left(t^{\mu\alpha}_{Q'a'}\,t^{\beta\nu}_{bQ} \pm t^{\mu\alpha}_{Q'b'}\,t^{\beta\nu}_{aQ}\right)$$
$$\mathsf{G}^{\mu\alpha,\beta\nu}_\pm(a,b) := \tfrac{1}{2}\left(\varepsilon^{\mu\alpha}_{Q'a'}\,t^{\beta\nu}_{bQ} \pm t^{\mu\alpha}_{Q'b'}\,\varepsilon^{\beta\nu}_{aQ}\right)$$
to structures independent of Q, Q':

$\delta^{\alpha\beta}$	$p^\alpha\gamma^\beta + \gamma^\alpha p^\beta$
$\delta^{\alpha\beta}\,\not{p}$	$p^\alpha\gamma^\beta - \gamma^\alpha p^\beta$
$[\gamma^\alpha,\gamma^\beta]$	$[p^\alpha\gamma^\beta + \gamma^\alpha p^\beta,\not{p}]$
$[\gamma^\alpha,\gamma^\beta,\not{p}]$	$[p^\alpha\gamma^\beta - \gamma^\alpha p^\beta,\not{p}]$
	$p^\alpha p^\beta$
	$p^\alpha p^\beta\,\not{p}$

- obtain 16 quadratic, 40 cubic 16 quartic terms $\Rightarrow$ **72 in total** √
- no kinematic singularities √

- **Transverse onshell basis:** GE, Fischer, PRD 87 (2013) & PoS Conf. X (2012)

$\mathsf{E}_+(P,P)$ $(++)$	$\widetilde{\mathsf{E}}_+(P,P)$ $(-+)$
$\mathsf{F}_+(P,P)$ $(++)$	$\widetilde{\mathsf{F}}_+(P,P)$ $(-+)$
$\mathsf{G}_+(P,P)$ $(++)$	$\widetilde{\mathsf{G}}_+(P,P)$ $(--)$
$\mathsf{G}_-(P,P)$ $(--)$	$\widetilde{\mathsf{G}}_-(P,P)$ $(++)$

$\mathsf{F}_+(P,Q)$ $(-+)$	$\widetilde{\mathsf{F}}_+(P,Q)$ $(++)$
$\mathsf{G}_+(P,Q)$ $(-+)$	$\widetilde{\mathsf{G}}_+(P,Q)$ $(+-)$
$\mathsf{F}_-(P,Q)$ $(+-)$	$\widetilde{\mathsf{F}}_-(P,Q)$ $(--)$
$\mathsf{G}_-(P,Q)$ $(+-)$	$\widetilde{\mathsf{G}}_-(P,Q)$ $(-+)$
$\mathsf{F}_+(Q,Q)$ $(++)$	$\widetilde{\mathsf{F}}_+(Q,Q)$ $(-+)$

VCS
VVCS
RCS
Scalar vertex

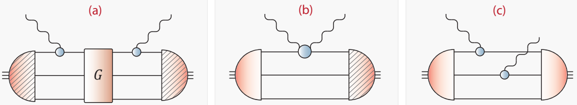
- Simple
- analytic in all limits
- manifest crossing and charge-conjugation symmetry
- scalar & pion pole only in a few Compton form factors
- Tarrach's basis can be cast in a similar form

Compton amplitude at quark level

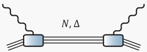
Baryon's **Compton scattering amplitude,** consistent with Faddeev equation:
GE, Fischer, PRD 85 (2012)

$$\langle H|J^\mu J^\nu|H\rangle = \bar{\chi}\left(G^{-1^\mu} G\, G^{-1^\nu} + G^{-1^\nu} G\, G^{-1^\mu} - (G^{-1})^{\mu\nu}\right)\chi$$

In rainbow-ladder (+ crossing & permutation):

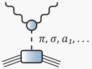


- **Born (handbag) diagrams:** G = **1** + T
- all s- and u-channel **nucleon resonances:**



1PI **quark 2-photon vertex:**

all t-channel **meson poles**

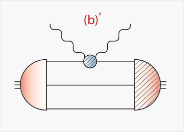


cat‘s ears diagrams

√ crossing symmetry
√ em. gauge invariance
√ perturbative processes included
√ s, t, u channel poles generated in QCD

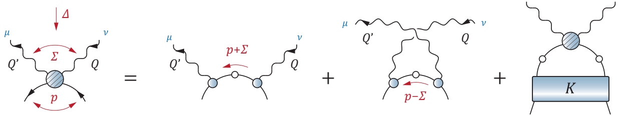
Compton amplitude at quark level

Collect all (nonperturbative!) **'handbag' diagrams**, where photon couples to same quark:
no nucleon resonances, no cat's ears



- **not electromagnetically gauge invariant**, but comparable to 1PI ‚structure part' at nucleon level?
- reduces to **perturbative handbag** at large photon momenta
- but also all **t-channel poles** included!

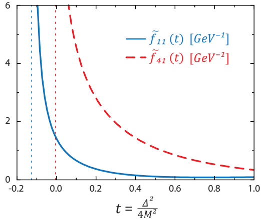
Represented by full **quark Compton vertex**, including Born terms.
Satisfies inhomogeneous BSE:



Solved in rainbow-ladder: 128 tensor structures (72 transverse).
Simplifies dramatically by choice of convenient basis!

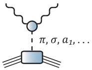
Compton amplitude at quark level

Quark Compton vertex:
recovers t-channel poles,
e. g. **scalar** and **pion** √

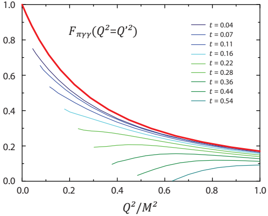


GE & Fischer, PRD 87 (2013)

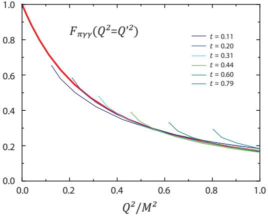
Quark Compton vertex and **nucleon Compton amplitude:** residues at pion pole recover $\pi\gamma\gamma$ **transition form factor** √



Rainbow-ladder result:
Maris & Tandy, PRC 65 (2002)



(extracted from
quark Compton vertex)



(extracted from
nucleon Compton amplitude)

Fermion Compton vertex

2-photon WTI $\Rightarrow$ general **offshell fermion Compton vertex** can be written as

$$\Gamma^{\mu\nu} = \Gamma^{\mu\nu}_{\mathrm{B}} + \Gamma^{\mu\nu}_{\mathrm{BC}} + \Gamma^{\mu\nu}_{\mathrm{T}} + \Gamma^{\mu\nu}_{\mathrm{TT}}$$

Born **WTI** **WTI-T**

- 2-photon equivalent of **Ball-Chiu vertex,** fixed by quark propagator & quark-photon vertex
- no kinematic singularities

Transverse

- not constrained by WTI, calculated from BSE
- contains **t-channel poles**
- **no kinematic singularities**
- 72 elements offshell **(18 elements onshell)**

General **structure** of fermion two-photon vertex (both offshell and onshell) determined.

- Onshell amplitude: gauge-invariant separation!
- Quark Compton vertex: all these will contribute to Compton form factors ($\Rightarrow$ polarizabilities, structure functions, GPDs, etc.). Dominant contributions?

$\Rightarrow$ Born **(handbag)?**

$\Rightarrow$ WTI, WTI-T **(em. gauge invariance) ?**

$\Rightarrow$ Fully transverse part **(t-channel poles) ?**

Summary

So far:

- Structure analysis of **nucleon Compton amplitude** & **quark Compton vertex**
- Nonperturbative calculation of **handbag part** (quark Compton vertex = Born + t-channel), t-channel pole behavior reproduced.

Next:

- Extract **polarizabilities** (subtraction needed to restore gauge invariance)
- **Two-photon exchange** contribution to form factors
- **GPDs & nucleon PDFs**
- Study **offshell effects** at nucleon level

Long term:

- **Improve truncations** (pion cloud, decay channels, quark six-point function)
- Access larger **phase space** (e.g. timelike region in $p\bar{p} \to \gamma\gamma$)

Thanks for your attention.

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