Consequences of a dressed quark-gluon vertex in heavy-light mesons

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P25121-N27

Jefferson Lab, May 21, 2014,

Motivation

- QCD and the structure of hadrons

  ⋆ the role of the quark masses in hadron spectrum and structure

- Heavy-light mesons

  ⋆ Dynamics of the heavy-light meson controlled by dynamics of the heavy quark

  ⋆ For mesons with $m_Q \sim m_M$ and $\frac{m_{\bar{q}}}{m_Q} \sim 0$: some conditions are provided by "heavy-quark symmetry"

  Neubert Phys. Rept. 245 (1994)

- Heavy-quark symmetry as a guidance

  ⋆ Use constraints of heavy-quark symmetry to check the correct behaviour of the mass

  ⋆ Similar procedure within point-form Hamiltonian dynamics using a constituent quark model: fully relativistic treatment of light quark is necessary

  Gomez-Rocha, Schweiger PRD 86 (2012)

- Limitations of BSE/DSE description of hadrons when using the rainbow-ladder truncation approach

  ⋆ In particular: difficulties to describe heavy-light mesons

The main goal of this work...

Based on previous studies of consequences of a dressed quark-gluon vertex in $q\bar{q}$ mesons...

Bender, Detmold, Thomas, Roberts, PRC **65** (2002)
Bhagwat, Höll, Krassnigg, Roberts, Tandy PRC **70** (2004)

Use the same interaction model:

Munczek and Nemirovsky PRD **28**, 181 (1983)

- sufficiently simple → allow to compute an arbitrary number of iterations
- sufficiently realistic → important features in common with QCD: e.g. confinement, dynamical chiral symmetry breaking

... attempt to generalize to $q\bar{Q}$ mesons
... ask:

Which kind of consequences do corrections to the bare vertex in heavy-light mesons have?

or...

Which kind of heavy-light physics are we "truncating" using RL approximation?

Dyson-Schwinger/Bethe-Salpeter approach

The interaction model

DSE:

$$S^{-1}(p) = i\not{p} + \mathbf{1}m_q + \int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \Gamma^a_\nu(q;p)$$

Interaction model : $\mathcal{D}_{\mu\nu}(k) := \frac{4}{3}\delta_{\mu\nu}(2\pi)^4 \mathcal{G}^2 \delta^4(k)$

Munczek and Nemirovsky PRD **28**, 181 (1983)

DSE becomes an algebraic equation

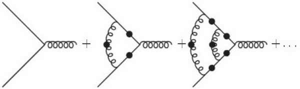
$\Gamma^{\mathcal{C}}_\mu(q;p) =$ 

figure adapted from Bender et *al.* PRC **65** 2002

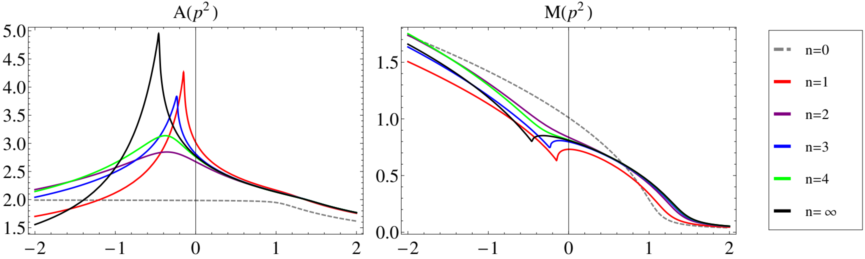
- ⇒ Solvable at every truncation order: $n = 0, 1, 2, 3...$
- ⇒ Analytical solution in RL
- ⇒ Solvable in the limit $n \to \infty$ (fully dressed)

$$\Gamma^{\mathcal{C}}_{\mu,n+1}(p) = -\mathcal{C}\gamma_\nu S(p)\Gamma^{\mathcal{C}}_{\mu,n}(p)S(p)\gamma_\nu$$

Solutions to the DSE for n loops

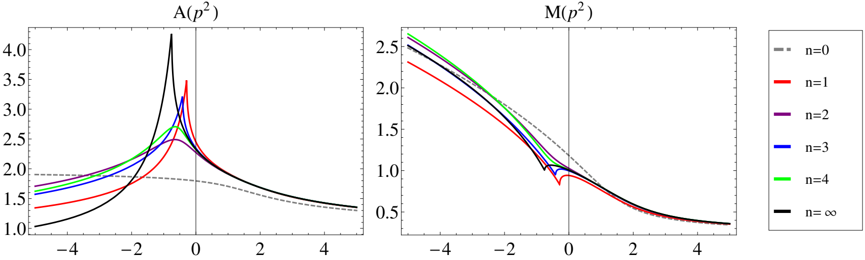
for $\mathbf{m_u = 10}$ MeV:

Bhagwat et *al.* PRC **70** (2004)



for $\mathbf{m_s = 0.166}$ GeV:

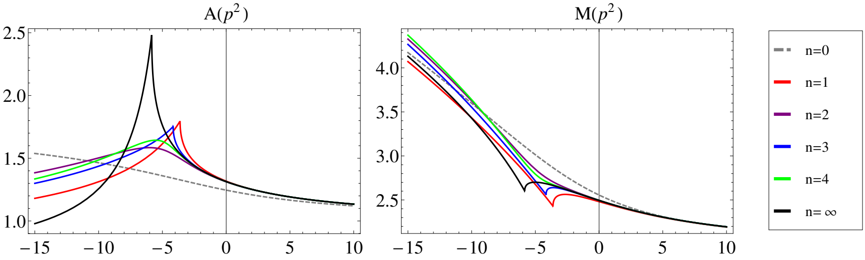
$C = 0.51$



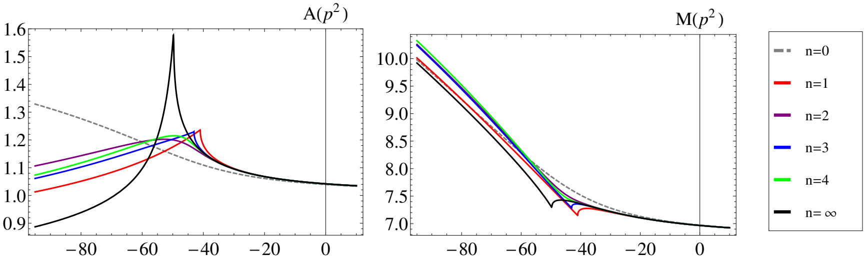
Quark propagator: $S^{-1}(p) = i\not{p}A(p^2) + B(p^2)$; $M(p^2) := B(p^2)/A(p^2)$

Solutions to the DSE for n loops

for $\mathbf{m_c} = \mathbf{1.33}$ GeV: $C = 0.51$



for $\mathbf{m_b} = \mathbf{4.62}$ GeV:



Quark propagator: $S^{-1}(p) = i\not{p}A(p^2) + B(p^2)$; $M(p^2) := B(p^2)/A(p^2)$

Dyson-Schwinger/Bethe-Salpeter approach

The interaction model

BSE:

$$\mathbf{\Gamma}_M(p;P) = \int \frac{d^4q}{(2\pi)^4} g^2 [\mathcal{K}(p;q;P)] \underbrace{S(q_+)\mathbf{\Gamma}_M(q;P)S(q_-)}_{\chi_M(q;P)} \Gamma^a_\nu(q;p)$$

$$p_+ = \eta P, \quad p_- = (\eta - 1)P$$

Use an appropriate kernel
that guarantees validity of Ward-Takahashi identities

Bender et *al.* PRC**65**(2002)

Generalize to $m_q \neq m_{\bar{q}}$ and use our MN model to simplify the calculation

Dyson-Schwinger/Bethe-Salpeter approach

The interaction model

Generalize to $m_q \neq m_{\bar{q}}$

$$\mathbf{\Gamma}_M(p;P) = \int_q^\Lambda \mathcal{D}_{\mu\nu}(p-q)\, l^a \frac{1}{2}\big[\gamma_\mu \chi_M(q;P)\, l^a\, \Gamma_\nu(q_-,p_-) + \Lambda^a_{M\nu}(q,p;P) S(q_-)\gamma_\mu + l^a\, \Gamma_\nu(q_+,p_+)\chi_M(q;P)\gamma_\mu + \gamma_\mu S(q_+)\,\Lambda^a_{M\nu}(q,p;P)\big],$$

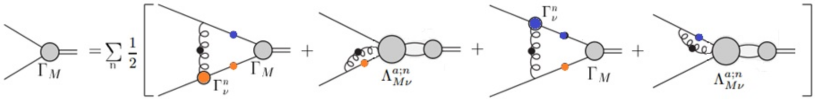


figure adapted from Bender et *al.* PRC **65** (2002)

Derive recursion relation for $\Lambda^a_{M\nu}(q,p;P)$ for $m_q \neq m_{\bar{q}}$

$\Rightarrow$ Corrections are more involve than for $m_q = m_{\bar{q}}$

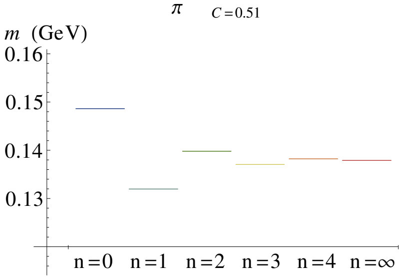
Search for bound-state solutions...

- Pseudoscalar mesons: $q\bar{q}$, $Q\bar{Q}$, $q\bar{Q}$
- Vector mesons: $q\bar{q}$, $Q\bar{Q}$, $q\bar{Q}$
- Scalars $\to$ no solution
- Axial vectors $\to$ no solution

Solutions to the BSE

First impression for $q\bar{q}$ systems

The pion



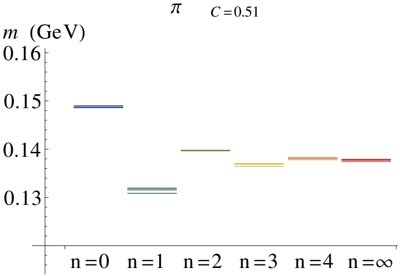
n = # of loops

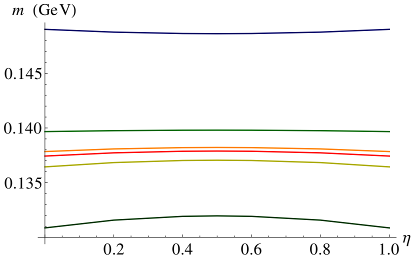
Now: check η-dependence, since this is an issue for unequal-mass constituents
(see also original MN-paper, η was used as fitting parameter).

$$\eta = \frac{p_+}{P}, \quad (\eta - 1) = \frac{p_-}{P}$$

Why η-dependence?

Because MN omits 2 of 4 covariants in BS amplitude $\rightarrow$ this breaks Lorentz covariance: systematic error $< 0.8\%$

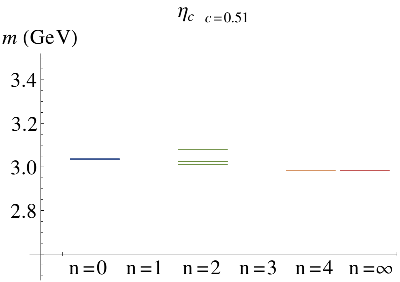




Correction to RL $\sim 8\%$ for $\eta = 0.5$

Heavy quarkonia $Q\bar{Q}$

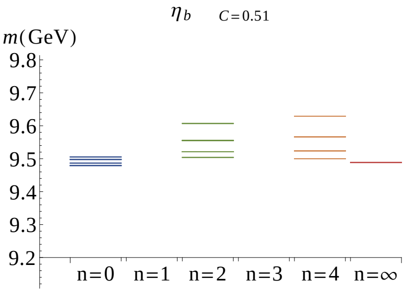
Charmonium



Correction to RL $\sim 1.6\%$ for $\eta = 0.5$

Error due to η-dep $< 6\%$

Bottomonium



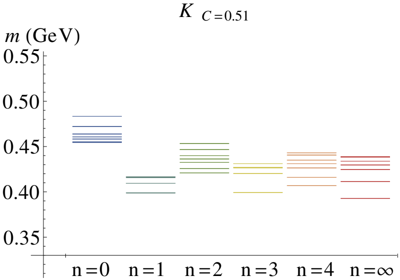
Correction to RL $\sim 0.2\%$ for $\eta = 0.5$

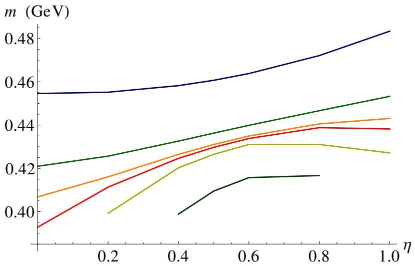
Error due to η-dep $< 1.5\%$

η-dep increases with increasing n

First check for mesons with $m_q \neq m_{\bar{q}}$:

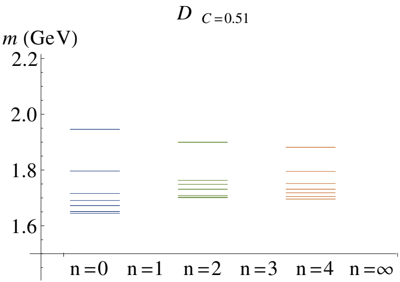
Kaon

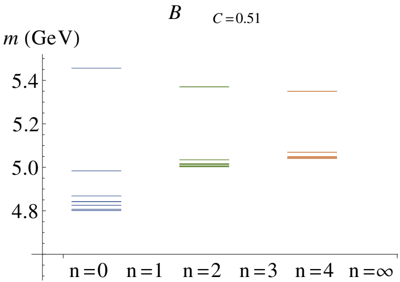




Correction to RL $\sim 7.2\%$ for $\eta = 0.5$
Error due to η-dep $< 10\%$

$q\bar{Q}$-mesons





Correction to RL $\sim 6.3\%$ for $\eta = 0.75$
Error due to η-dep $< 15\%$
$m_D^{\rm full} = 1.68$ GeV

Correction to RL $\sim 3.6\%$ for $\eta = 0.95$
Error due to η-dep $< 12\%$
$m_B^{\rm full} = 5.08$ GeV

η-dep decreases with increasing n

Summary

- Interest in $q\bar{Q}$ meson properties, phenomenology and structure from QCD
- Showed you an approach to these problems within DSE/BSE formalism
- Based on previous works, used an interaction model where
  - ⋆ Very heavy quarks and mesons can be studied numerically (even heavy-quark limit)
  - ⋆ Effects beyond the popular RL truncation of DSE/BSE can be studied systematically and quantitatively

$$\pi \sim 8\% \qquad K \sim 7.2\ \%$$
$$\eta_c \sim 1.6\% \qquad D \sim 6.3\ \%$$
$$\eta_b \sim 0.2\% \qquad B \sim 3.6\ \%$$

BUT: η-dependence must be better understood

Conclusions and Outlook

- Setup established previously was adapted to investigate $q\bar{q}$, $Q\bar{Q}$, $q\bar{Q}$ meson
- Corrections to the RL truncation are more complicated in $q\bar{Q}$ mesons than in $q\bar{q}$ mesons
- Model artifacts have to be taken into account and kept under control (η-dependence)
- Heavy-light meson masses computed from the given model and parameters are reasonable
- Check heavy-quark symmetry predictions: relations between vector and pseudoscalar mesons in extreme case $m_Q \gg m_q$
- Decay constants

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Thank you!