MESON PHENOMENOLOGY IN A BETHE-SALPETER EQUATION APPROACH

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LIGHT CONE, 29 MAY 2014

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FWF

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Project FWF-P25121-N27

www.ModelsofHadrons.com

Motivation

- **Goal**: develop a model to study hadrons that is applicable from chiral limit up to $m_q = m_b$

- QCD Green's functions
  - connected to confinement
  - running coupling
  - ingredients for hadron phenomenology

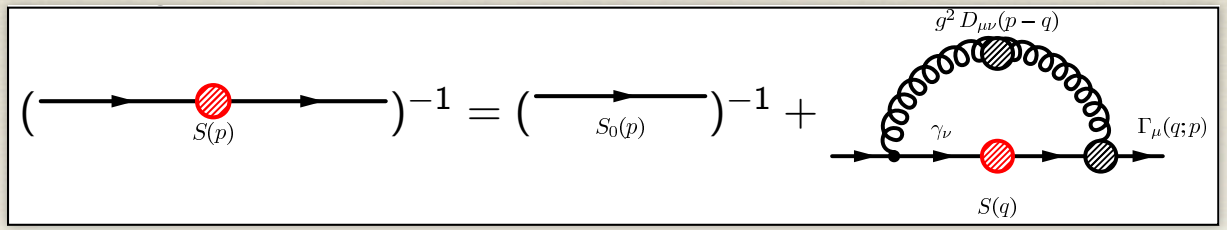
- **Means**: Dyson-Schwinger and Bethe-Salpeter Equations
  - All momentum scales
  - Both heavy and light (also DCSB)

- **Setup**
  - sophisticated Landau gauge effective interaction
  - simple setup (rainbow ladder truncation)

Motivation

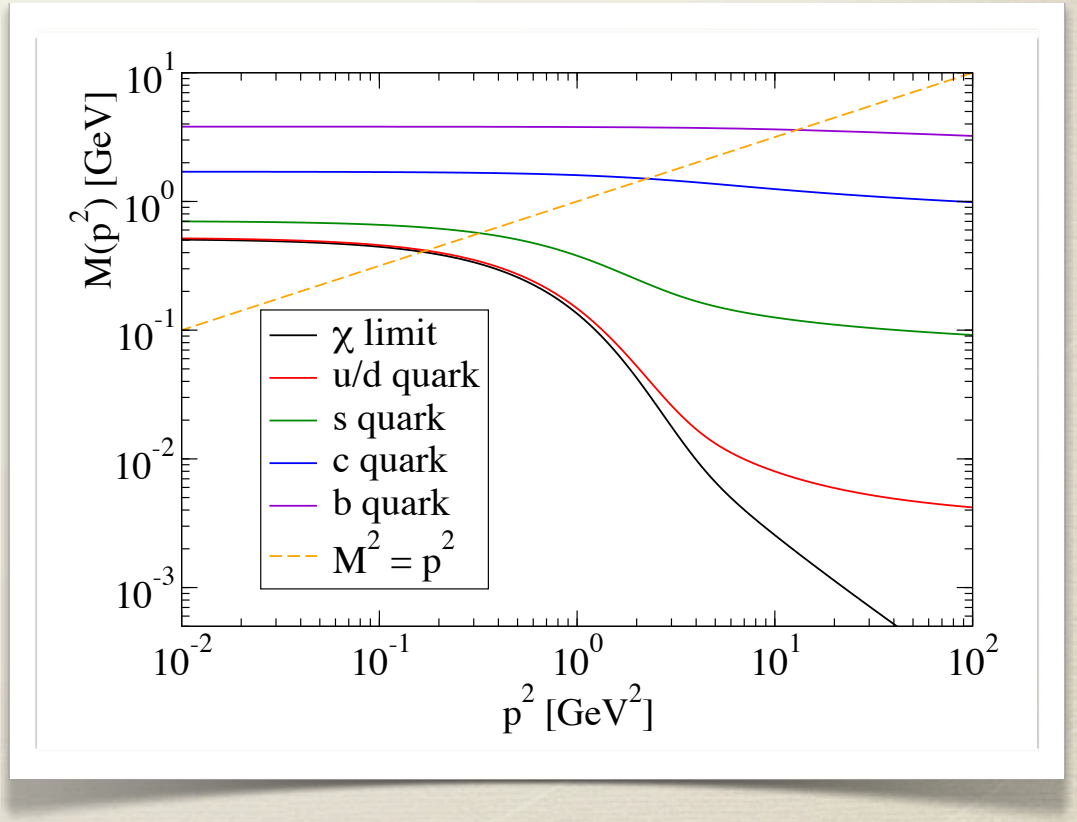
- **Calculate observables**
  - masses, splittings
  - excitations of both radial and angular quantum numbers
  - form factors, decays
  - exotic states **[T. Hilger]**
  - multiquark states
  - heavy-light systems **[M. Gomez-Rocha]**
- **Plan**
  - start by describing bottomonium spectrum, continue to charmonium
  - adjust the effective interaction
- So far: bottomonium spectrum agrees very well with experiment

Gap Equation

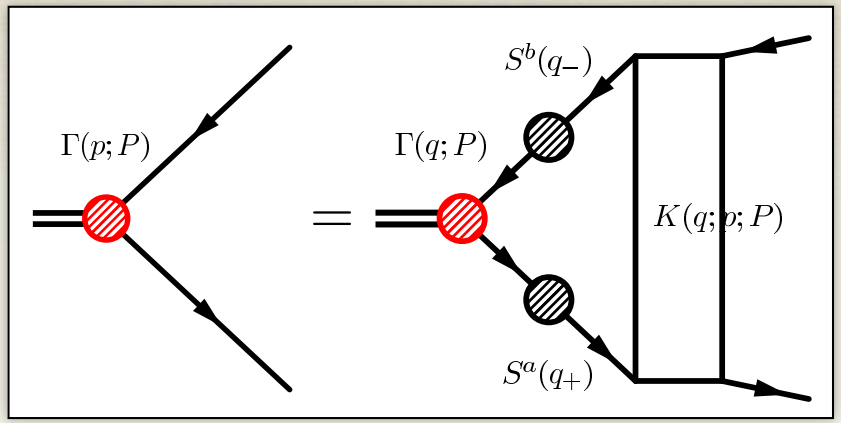


$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

- Perturbation theory reproduced order by order via weak coupling expansion
- Get nonperturbative solution (mass function $M(p^2)$)
- DCSB



Homogeneous Bethe-salpether Equation



$$\Gamma(p;P) = \int_q S^a(q+\eta P)\,\Gamma(q;P)\,S^b(q-(1-\eta)P)\,K(q,p;P)$$

- Bound state mass $P^2 = -M^2$
- Input:
  - dressed quark propagator ***S(q)*** (from the gap equation)
  - quark-antiquark scattering kernel ***K(q,p;P)*** (one gluon exchange)
  - ***Effective quark-gluon interaction*** employed in both DSE and BS kernel

Rainbow Ladder truncation

- DSE/BSE build up an infinite system of coupled integral eqs
  $\Rightarrow$ ***truncate the system***
- RL approximation:
  - *Rainbow* truncation for the gap equation
  - *Ladder* truncation for the BSE
- Ward-Takahashi identity satisfied
  - Bare quark-gluon vertex
  - Bare gluon propagator
  - Effective coupling multiplies the product qgv and gp (modeling)
- Well suited for heavy quarks (e.g. Coulomb gauge studies)
- Successfully employed in earlier investigations of meson spectra in Landau gauge **Blank, Krassnigg PRD84 2011; Krassnigg PRD80 2009**

EFFECTIVE INTERACTION

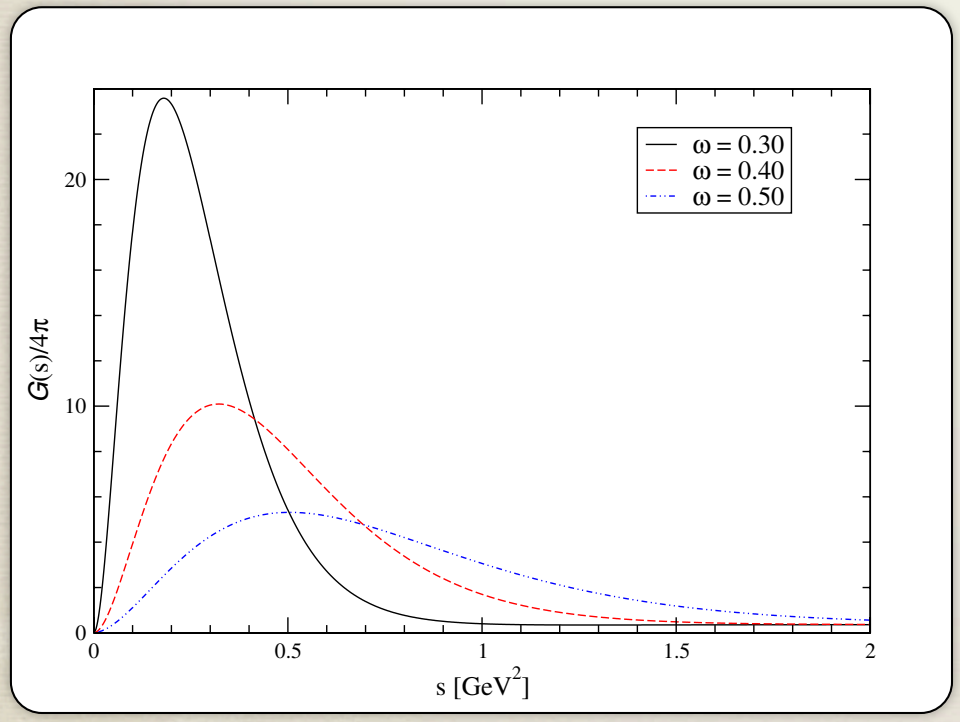
- **Requirements**
  - UV regime determined by perturbative QCD
  - Break chiral symmetry dynamically ⇒ IR enhancement
  - Fit pion mass and decay constant

    ⇒ effective strength is essential
  - Ansatz at low and intermediate momentum
- **Model**
  - P. Maris, P. C. Tandy: **[Maris, Tandy, PRC 60 1999]** and following series of papers
  - Successful description of light pseudoscalar and vector mesons
  - Charge radii, strong and radiative decay etc.

EFFECTIVE INTERACTION

$$\mathcal{D}(p^2) = \frac{D}{2}\,\frac{(2\pi)^2}{\omega^6}\,p^2 e^{-p^2/\omega^2} + \mathcal{F}_{UV}(p^2)$$

$$K(p;q;P) \Rightarrow \gamma_\nu \mathcal{D}((p-q)^2) D^{free}_{\mu\nu}(p-q)\gamma_\mu$$



[Krassnigg PRD 80 2009]

- preserves confinement
- gives the right amount of χSB
- provides the correct one-loop renormalization group behaviour of QCD via $\mathcal{F}_{UV}$
  - intermediate momentum range relevant for phenomenology
  - parameters: width ω, strength D

Nummerical solutions: DSE

$$S^{-1}(p) = S_0^{-1}(p) + \frac{4}{3}\int \frac{d^4q}{(2\pi)^4}\, g^2\, D_{\mu\nu}(p-q)\,\gamma_\nu\, S(q)\,\Gamma_\mu(q;p)$$

- Coupled nonlinear integral equations for quark propagator dressing functions

$$S^{-1}(p) = i\,\gamma\cdot p\, A(p^2) + B(p^2)$$

- Solve iteratively on the real p^2 axis
- On-shell condition $P^2 = -M^2$ leads to imaginary momentum

  $\Rightarrow$ analytical continuation of $A(p^2)$ and $B(p^2)$ into complex plane

Nummerical solutions: BSE

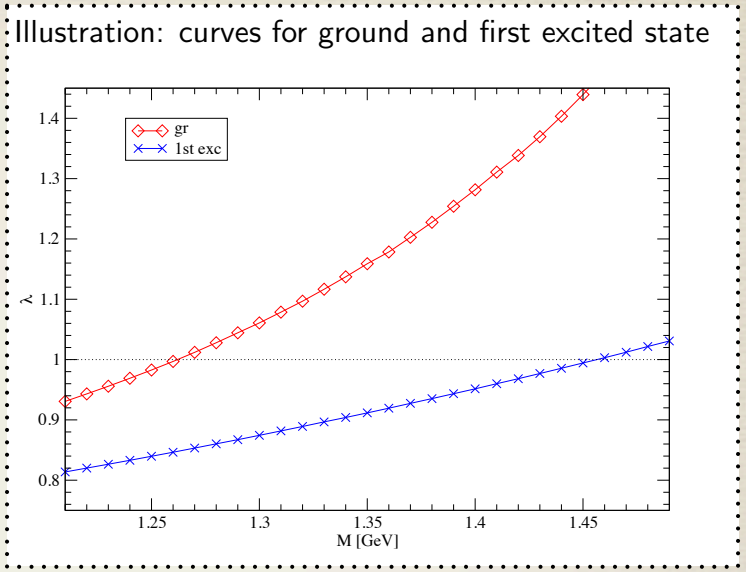
- Recast BSE into an eigenvalue problem and solve it for the largest few eigenvalues

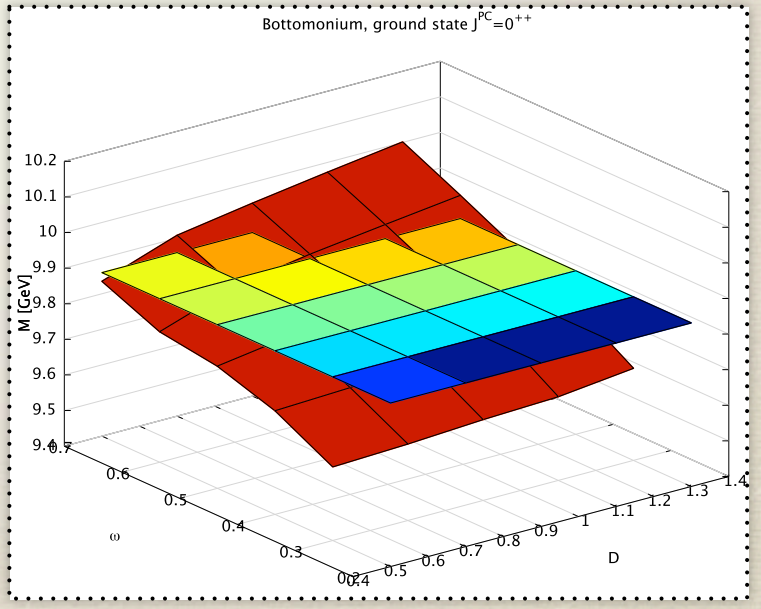
$$\lambda(P^2)\Gamma(p;P) = \tilde{K}(p;q;P)\Gamma(q;P)$$

- Obtain curves for the chosen eigenvalues
- Mass determined from the condition

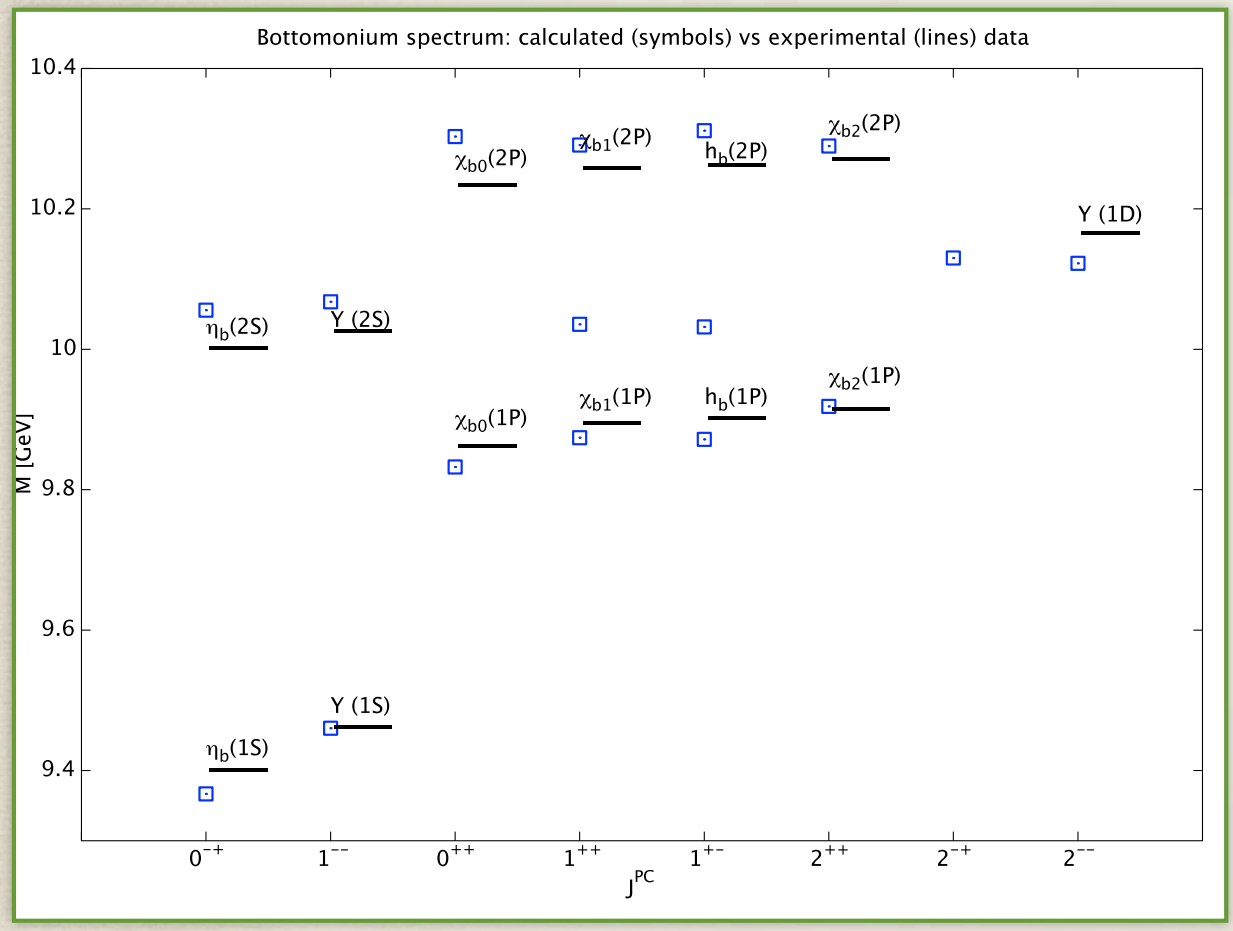
$$\lambda(P^2 = -M^2) = 1$$

- Read off the optimal set of parameters by intersecting the surface determined by (ω, D) with the experimental value (horizontal plane)
- $\omega \cdot D$ not necessarily constant





Bottomonium spectrum (Preliminary)



- Ground states and radial excitations
- Vector channel ground state fitted to experiment
- Good agreement with available experimental data
- Unclear whether all excitations appearing as solutions of BSE are physical $\Rightarrow$ spurious states?

Summary

- Comprehensive RL truncated model based on DSE/BSE
- Thorough analysis of the effective interaction
- Gap equation: solve iteratively on the real axis and perform analytical continuation in the complex plane
- BSE: Solve eigenvalue problem and obtain meson masses from the eigenvalues
- Bottomonium spectrum agrees remarkably well with experiment (still need to understand spuriosities)
- Charmonium in preparation

Outlook

- Fix the parameters of the interaction (quark mass dependence?)
- Meson states with exotic quantum numbers
- Hadronic, leptonic, electromagnetic transitions (from the BS amplitudes, i.e. eigenvectors of BSA)
- **Other directions:**
  - Finite temperature and chemical potential
  - Baryons