Medium Modifications of Mesons*

chiral symmetry restauration, QCD sum rules for medium modificated D and ρ mesons, and Bethe-Salpeter equations

Thomas Hilger

- Medium modifications of D mesons $\leftrightarrow$ chiral condensate
- Chiral partner sum rules $\leftrightarrow$ chiral condensate
- Impact of chirally odd condensates on the ρ meson
- Meson masses from Dyson-Schwinger – Bethe-Salpeter eqs.

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* with B. Kämpfer, S. Leupold, S. Dorkin, L. Kaptari

supported by BMBF & GSI-FE

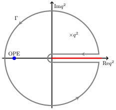
Hadron physics and QCD sum rules

Current-Current Correlator

$$\Pi^{\mathrm{X}}_{\mu\nu}(q) = i\int d^4x e^{-iqx}\langle \mathrm{T}\left[j^{\mathrm{X},\tau}_{\mu}(x)\left(j^{\mathrm{X},\tau}_{\nu}(0)\right)^{\dagger}\right]\rangle$$

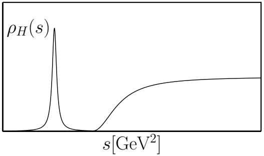
dispersion relation

$$\Pi(q^2) = \frac{1}{\pi}\int_0^\infty \mathrm{d}s \frac{\Delta\Pi(s)}{s-q^2}$$



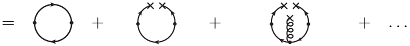
separation of energy scales

hadronic properties encoded in spectral density



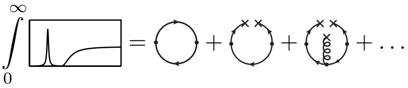
Operator Product Expansion

$$= C_1(q) + C_2(q)\langle\bar{q}q\rangle + C_3(q)\langle\bar{q}g\sigma\mathscr{G}q\rangle + \dots$$



QCD condensates

- medium dependence
- order parameter of chiral symmetry



Probing chiral symmetry restoration via the chiral condensate - light quark currents

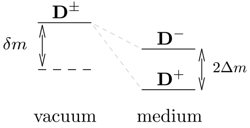
Example ρ meson: $j^{\rho}_{\mu}(x) = \bar{\psi}\frac{\sigma_3}{2}\gamma_\mu\psi = \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)(x)$

$$\Pi^{\rho}_{\mu\nu}(q) = \frac{i}{4}\int \mathrm{d}^4x e^{iqx}\langle \mathrm{T}\left[(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)(x)(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)(0)\right]\rangle$$

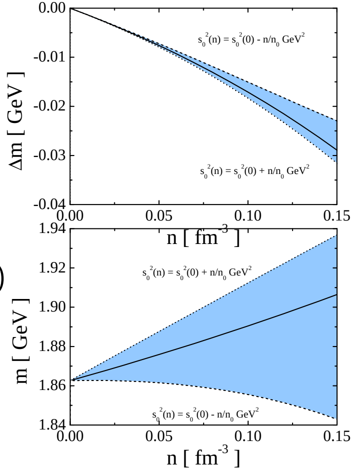
- $\langle\bar{q}q\rangle$ suppression in light-quark meson operator product expansion (e.g. ρ meson sum rules): $m_q\langle\bar{q}q\rangle$
- $\langle\bar{q}q\rangle$ influence via assumptions/models: e.g.
    - $\langle\bar{q}\Gamma q\bar{q}\Gamma q\rangle \propto \langle\bar{q}q\rangle^2$
      $\rightarrow$ fragile transition to medium
    - continuum threshold $s_0 \leftrightarrow f_\pi \leftrightarrow \langle\bar{q}q\rangle$
- determination of other order parameters (e.g. four-quark condensates $\langle\bar{q}\Gamma q\bar{q}\Gamma q\rangle$)?

Probing chiral symmetry restoration via the chiral condensate - heavy-light quark currents*

- $\langle \bar{q}q \rangle$ amplification due to heavy quark mass, e.g. D meson sum rules: $m_c \langle \bar{q}q \rangle$



- mass splitting is sensitive to:
  $\langle q^\dagger q \rangle$ ($= \frac{3}{2} n \propto$ net quark density)
  $\langle \bar{q}q \rangle$, $\langle q^\dagger g \sigma \mathcal{G} q \rangle$
- mass center $\mathrm{D} - \bar{\mathrm{D}}$ is sensitive to
  - $\langle \bar{q}q \rangle$
  - continuum threshold



*[TH, R. Thomas, B. Kämpfer, Phys. Rev. C 79: (2009) 025202]
[S. Zschocke, TH, B. Kämpfer, Eur. J. A 47: (2011) 151]

Light-quark chiral partner sum rules

- $n_f = 2$ Lagrangian:

$$\mathscr{L} = \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}^T \left(i\gamma_\mu \partial^\mu - \begin{bmatrix} m_u & 0 \\ 0 & m_d \end{bmatrix} \right) \begin{pmatrix} u \\ d \end{pmatrix}$$

for $m_{u,d} = 0$ invariant under transformation

$$\psi = \begin{pmatrix} u \\ d \end{pmatrix} \to e^{-i\gamma_5 \frac{\vec{\lambda}}{2} \vec{\Theta}} \psi$$

$$j_\mu^{\mathrm{V},\tau}(x) = \bar{\psi}\gamma_\mu \tau \psi \longrightarrow j_\mu^{\mathrm{A},\tau}(x) = \bar{\psi}\gamma_5 \gamma_\mu \tau \psi$$

- chirally symmetric ground state → current-current correlators

$$\Pi_{\mu\nu}^{\mathrm{X}}(q) = i \int d^4 x e^{-iqx} \langle \mathrm{T} \left[j_\mu^{\mathrm{X},\tau}(x) \left(j_\nu^{\mathrm{X},\tau}(0) \right)^\dagger \right] \rangle$$

are "blind" to parity

Weinberg-Kapusta-Shuryak sum rules*

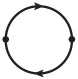
- finite density/temperature sum rules for vector–axial-vector currents of massless quarks

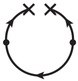
$$\int_0^\infty \frac{ds}{s} \Delta\Pi^{\mathrm{V-A}} = F_\pi^2$$

$$\int_0^\infty ds \Delta\Pi^{\mathrm{V-A}} = 0$$

$$\int_0^\infty ds\, s \Delta\Pi^{\mathrm{V-A}} = -2\pi \langle \alpha_s O_\mu^\mu \rangle$$

- chiral condensate suppressed by light quark mass





* [S. Weinberg, Phys. Rev. Lett. 18 (1967) 507]
[J. Kapusta, E. Shuryak, Phys. Rev. D49 (1994) 4694]

Chiral partner sum rules for heavy-light mesons

- $n_f = 3$ Lagrangian with a "non-light" quark:

$$\mathscr{L} = \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{h} \end{pmatrix}^T \left(i\gamma_\mu \partial^\mu - \begin{bmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_h \end{bmatrix} \right) \begin{pmatrix} u \\ d \\ h \end{pmatrix}$$

for $m_{u,d} = 0$ invariant under transformation

$$\psi = \begin{pmatrix} u \\ d \\ h \end{pmatrix} \to e^{-i\gamma_5 \frac{\vec{\lambda}}{2} \vec{\Theta}} \psi = \begin{pmatrix} u' \\ d' \\ h \end{pmatrix}$$

$$j_\mu^{\mathrm{V},\tau}(x) = \bar{\psi}\gamma_\mu \tau \psi \longrightarrow j_\mu^{\mathrm{A},\tau}(x) = \bar{\psi}\gamma_5 \gamma_\mu \tau \psi$$

Scalar and pseudoscalar mesons*

Moments of the spectral difference for spin-0 heavy-light currents $j^{\mathrm{P,S}} = \bar{q}_l(i\gamma_5)q_h$:

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\, \omega \Delta\Pi^{\mathrm{P-S}}(\omega) = -\,2m_h\langle\bar{q}q\rangle \,,$$

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\, \omega^3 \Delta\Pi^{\mathrm{P-S}}(\omega) = -\,2m_h^3\langle\bar{q}q\rangle + m_h\langle\bar{q}g\sigma\mathscr{G}q\rangle - m_h\,\langle\Delta\rangle \,,$$

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\, \omega^5 \Delta\Pi^{\mathrm{P-S}}(\omega) = -\,2m_h^5\langle\bar{q}q\rangle + 3m_h^3\langle\bar{q}g\sigma\mathscr{G}q\rangle - 3m_h^3\,\langle\Delta\rangle + \ldots \,.$$

- spectral difference driven only by order parameters of chiral symmetry breaking
- heavy quark mass amplifies influence of chiral condensate
- hierarchy of order parameters: $\langle\bar{q}q\rangle$,
  $\langle\bar{q}g\sigma\mathscr{G}q\rangle - \langle\Delta\rangle \propto \langle\bar{q}D_0^2q\rangle$

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*[TH, R. Schulze, B. Kämpfer, J. Phys. G: Nucl. Part. Phys. 37 (2010) 094054]
[TH, B. Kämpfer, Nucl. Phys. Proc. Suppl. 207-208 (2010) 277]
[TH, B. Kämpfer, S. Leupold, Phys. Rev. C84: (2011) 045202]
[TH, T. Buchheim, B. Kämpfer, S. Leupold, Prog. Part. Nucl. Phys., in print]

Vector and axialvector mesons

- currents not conserved
- longitudinal (L) and transversal (T) projection

$$\Pi_{\mu\nu}(q) = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\Pi_{\mathrm{T}}(q) + \frac{q_\mu q_\nu}{q^2}\Pi_{\mathrm{L}}(q)$$

- mixing of quantum numbers ($m_l \to 0$)

$$\Pi_{\mathrm{L}}^{\mathrm{V-A}}(q) = -\frac{m_h^2}{q^2}\Pi^{\mathrm{S-P}} - 2m_h\langle\bar{q}q\rangle$$

$$\Pi_{\mathrm{T}}^{\mathrm{V-A}}(q) = -\frac{m_h^2}{3q^2}\Pi^{\mathrm{P-S}}(q) - \frac{1}{3}g^{\mu\nu}\Pi_{\mu\nu}^{\mathrm{V-A}}(q) - \frac{2}{3}\frac{m_h}{q^2}\langle\bar{q}q\rangle$$

Moments of the spectral difference for spin-1 heavy-light currents:*

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\,\omega\,\Delta\Pi_{\mathrm{T}}^{\mathrm{V-A}}(\omega) = -\,2m_h\langle\bar{q}q\rangle\ ,$$

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\,\omega^3\Delta\Pi_{\mathrm{T}}^{\mathrm{V-A}}(\omega) = -\,2m_h^3\langle\bar{q}q\rangle - \frac{4}{3}m_h\langle\Delta\rangle\ ,$$

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\,\omega^5\Delta\Pi_{\mathrm{T}}^{\mathrm{V-A}}(\omega) = -\,2m_h^5\langle\bar{q}q\rangle + m_h^3\langle\bar{q}g\sigma\mathscr{G}q\rangle - \frac{11}{3}m_h^3\langle\Delta\rangle + \ldots\ .$$

- similar structure as in the P-S case
- order parameters: $\langle\bar{q}q\rangle$, $\langle\Delta\rangle$, $\langle\bar{q}g\sigma\mathscr{G}q\rangle$

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*[TH, B. Kämpfer, S. Leupold, Phys. Rev. C84: (2011) 045202]
[TH, T. Buchheim, B. Kämpfer, S. Leupold, Prog. Part. Nucl. Phys., in print]

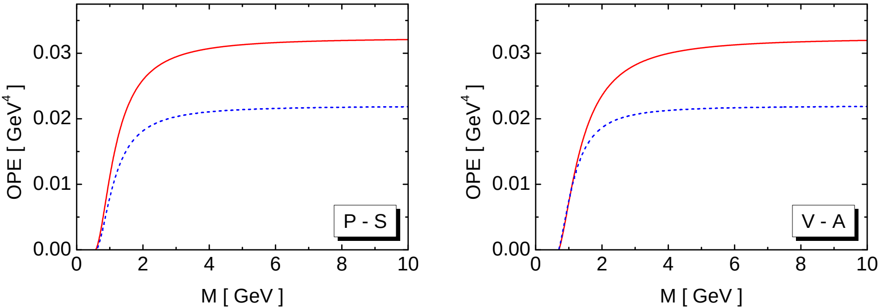
- Weinberg's first sum rule is recovered for $\tilde{\Pi}_{\rm T} = \Pi_{\rm T}/q^2$ (different analytic properties):

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega\omega\Delta\tilde{\Pi}_{\rm T}^{\rm V-A}(\omega) = 0$$

- heavy quark limit:

$$\Pi_{\rm T}^{\rm V-A}(q)\Big|_{m_2^2\gg|q^2|} \approx \Pi^{\rm P-S}(q)\Big|_{m_2^2\gg|q^2|} \approx -\frac{2}{m_2}\langle\bar{q}q\rangle$$

OPE for vacuum and medium



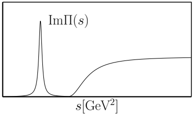
solid curve: vacuum

dashed curve: $n = n_0$

Chirally odd condensates in ρ meson QCD sum rules*

- ρ meson properties $\leftrightarrow$ dynamical chiral symmetry breaking

$$\frac{1}{\pi}\int_0^{s_+}\frac{\mathrm{d}s}{s}\,\Delta\Pi(s)\,e^{-s/M^2} = M^2 c_0\left[1-e^{-s_+/M^2}\right] + c_1 + \frac{c_2}{M^2} + \frac{c_3}{2M^4} + \frac{c_4}{6M^6}$$



$$\langle \mathcal{O}_4^{\mathrm{V}} \rangle \equiv \langle \mathcal{O}^{\mathrm{br}} \rangle + \langle \mathcal{O}^{\mathrm{sym}} \rangle$$

- $\langle \mathcal{O}_4^{\mathrm{V}} \rangle \overset{\text{factorization}}{\rightarrow} \langle \bar{q}q \rangle^2$ (order parameter)
- $\langle \mathcal{O}^{\mathrm{br}} \rangle \approx \frac{9}{7}\langle \bar{q}q \rangle^2$ [Bordes et al., JHEP 0602, 037 (2006)]

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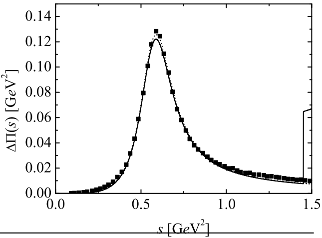
* [TH, R. Thomas, B. Kämpfer, S. Leupold, Phys. Lett. B709 (2012) 200]

The impact of chirally odd condensates on the ρ meson*

model independent moment of the spectral function

$$S \equiv \frac{\int_0^{s_+} \mathrm{d}s \Delta\Pi(s) e^{-s/M^2}}{\int_0^{s_+} \frac{\mathrm{d}s}{s} \Delta\Pi(s) e^{-s/M^2}}$$

- $S \stackrel{!}{=} m_\rho^2 = (776\,\mathrm{MeV})^2 \Longrightarrow \langle \mathcal{O}^{\mathrm{sym}} \rangle = (267\,\mathrm{MeV})^6$
  $\langle \mathcal{O}^{\mathrm{br}} \rangle = \langle \bar{q}q \rangle = 0 \Longrightarrow S \to S' = (660\,\mathrm{MeV})^2$
- $\mathrm{Im}\Pi \stackrel{!}{=} (\tau \to \nu + n\pi)$-data from ALEPH
  [Schael et al., Phys. Rep. 421, 191 (2005)]

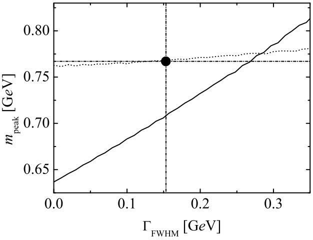


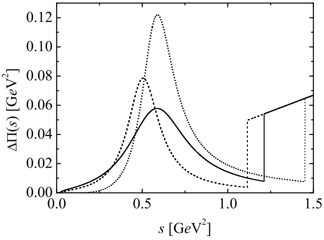
$\Longrightarrow \langle \mathcal{O}^{\mathrm{sym}} \rangle = (261\,\mathrm{MeV})^6$

*[TH, R. Thomas, B. Kämpfer, S. Leupold, Phys. Lett. B709 (2012) 200]

The impact of chirally odd condensates on the ρ meson*

- $\langle \mathcal{O}^{\mathrm{br}} \rangle = \langle \bar{q} q \rangle = 0$
- s-wave parametrization with one-pion threshold:
  $\Delta\Pi(s \le s_0) = \frac{F_0}{\pi} \frac{\sqrt{s}\Gamma(s)}{(s-m_0^2)^2 + s\Gamma^2(s)}$,
  $\Gamma(s) = \Theta(s - m_\pi^2)\Gamma_0 \sqrt{\left[1 - \frac{m_\pi^2}{s}\right] / \left[1 - \frac{m_\pi^2}{m_0^2}\right]}$





*[TH, R. Thomas, B. Kämpfer, S. Leupold, Phys. Lett. B709 (2012) 200]

The impact of chirally odd condensates on the ρ meson*

- comparison to NA60-data
  [Arnaldi et al., Eur. Phys. J C59 (2009) 607]
  [Arnaldi et al., Eur. Phys. J C61 (2009) 711]

  $\rightarrow$ vector meson dominance
  [van Hees, Rapp, Nucl. Phys. A806 (2008) 339]
  with $m = 830$ MeV, $\Gamma = 500$ MeV

- VOC scenario: $m_{\mathrm{peak}} = 830$ MeV $\rightarrow \Gamma_{\mathrm{FWHM}} = 380$ MeV

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* [TH, R. Thomas, B. Kämpfer, S. Leupold, Phys. Lett. B709 (2012) 200]

Dyson-Schwinger and Bethe-Salpeter

- Quark propagator (euclidean)

$$S_q^{-1}(p) = i\gamma\cdot p\, A(p) + B(p) = A(p)\,(i\gamma\cdot p + m(p))$$
$$= (i\gamma\cdot p\,\sigma_v(p) + \sigma_s(p))^{-1}$$

- homogeneous Bethe-Salpeter boundstate and Dyson-Schwinger equation (euclidean) in rainbow-ladder approximation

$$\Gamma(P,p) = -\frac{4}{3}\int \frac{d^4k}{(2\pi)^4}\,\gamma_\mu\, S(k_+)\Gamma(P,k)S(k_-)\,\gamma_\nu\,\left[g^2 D(p-l)\right]_{\mu\nu}$$

quark-gluon vertex in rainbow approximation

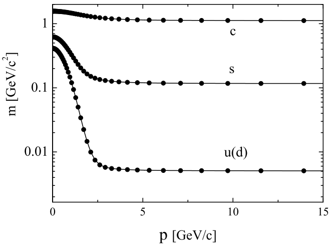
complex arguments $k_\pm = k + \eta_\pm P$, $\eta_+ + \eta_- = 1$

gluon propagator in ladder approximation

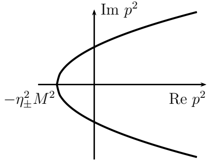
$$S_q^{-1}(p) = i\gamma\cdot p + \tilde{m} + \frac{4}{3}\int \frac{d^4l}{(2\pi)^4}\left[g^2 D(p-l)\right]_{\mu\nu}\gamma_\mu\, S_q(l)\,\gamma_\nu$$

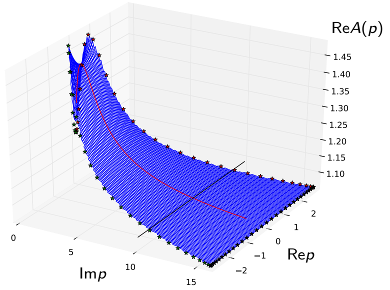
Dyson-Schwinger equation in the complex plane*

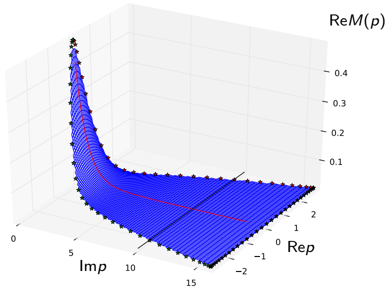
- Dyson-Schwinger equation easily solved along real axis:



- Dyson-Schwinger equation within the complex plane:

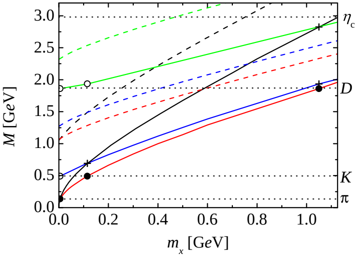


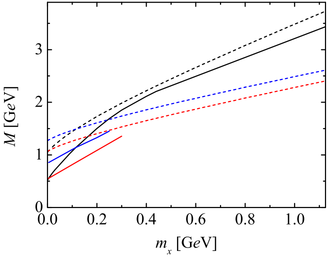




*[Dorkin et al., Few Body Syst. 49:247-254, 2011]

Resultate der Bethe-Salpeter–Gleichung*





* [S. Dorkin, TH, L. Kaptari, B. Kmpfer, Few Body Syst. **49** 247-254, 2011]

Summary I

QCD sum rules

1. mass splitting in heavy-light sector in next-to-leading order of density sensitive to chiral condensate

   [TH, R. Thomas, B. Kämpfer, Phys. Rev. C **79** (2009) 025202]
   [S. Zschocke, TH, B. Kämpfer, Eur. Phys. J. A **47** (2011) 151]
   [TH, B. Kämpfer, Nucl. Phys. B Proc. Suppl. **207-208** (2010) 025202]
   [TH, B. Kmpfer, Conf. Proc. Italian Phys. Soc. **99** (2010)]
   [B. Kmpfer, TH, H. Schade, R. Schulze, G. Wolf, PoSBormio **2010**]
   [R. Rapp et al., *In-medium excitations*, Lect. Notes Phys. **814** 335 (2011)]

2. chirale partner sum rules:
   chiral condensate $\times$ heavy quark mass dominates spectral difference of chiral partner

   [TH, B. Kämpfer, S. Leupold, Phys. Rev. C **84** (2011) 045202]
   [TH, T. Buchheim, B. Kämpfer, S. Leupold, Prog. Part. Nucl. Phys. **67** (2012) 188]
   [TH, R. Schulze, B. Kämpfer, J. Phys. G: Nucl. Part. Phys. **37** (2010) 094054]
   [TH, B. Kämpfer, Nucl. Phys. Proc. Suppl. **207-208** (2010) 277]

3. chiral symmetric sum rules require significant change of spectral moments

   [TH, R. Thomas, B. Kämpfer, S. Leupold, Phys. Lett. B **709** (2012) 200]

Summary II

Dyson-Schwinger–Bethe-Salpeter approach

3.

- coupled solution of Dyson-Schwinger and Bethe-Salpeter equation

  [S. Dorkin, TH, L. Kaptari, B. Kmpfer, Few Body Syst. **49** 247-254, 2011]
- investigation of analytic properties of the quark propagator

  [S. Dorkin, L. Kaptari, TH, B. Kmpfer, Phys. Rev. C **89** (2014) 034005]
- extension to finite densities/temperatures . . .