Der Wissenschaftsfonds.

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Quarkonia in a Bethe-Salpeter equation approach

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Schladming Winterschool
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Austrian Science Fund (FWF) project no. P25121-N27

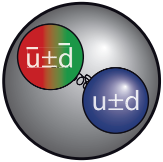
`Covariant.ModelsOfHadrons.com`

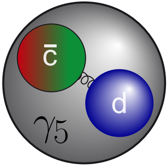
The Problem

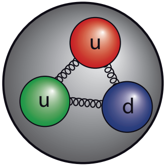
- study hadrons as composites of quarks and gluons
- issues
  - chiral symmetry and DχSB
  - Poincaré covariance
  - confinement
  - perturbative limit

$\rightarrow$ calculate observables

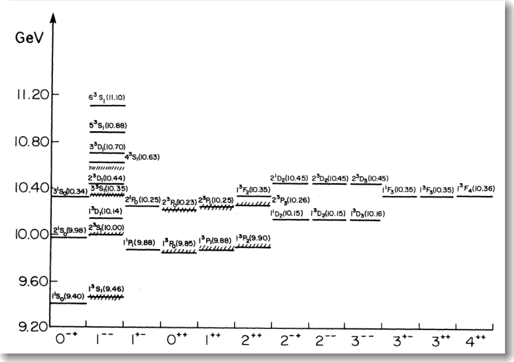
$\rightarrow$ comprehensive phenomenology







Motivation



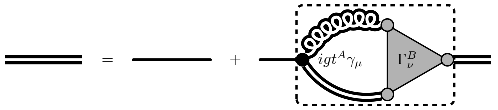
[Bottomonium by Godfrey, Isgur, 1985]

The Tool

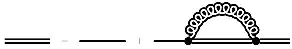
- Dyson-Schwinger equations can be used to solve QCD
- Bethe-Salpeter and Faddeev-type equations allow covariant and symmetry-preserving study of bound-state problems
- Infinite set of coupled (and nonlinear) integral equations
- Numerical studies: Truncation $\leftrightarrow$ numerical effort
- Make the truncation respect symmetries
- Construct sophisticated models
- Perform reliable calculations of hadron properties

Dyson-Schwinger and Bethe-Salpeter Equations

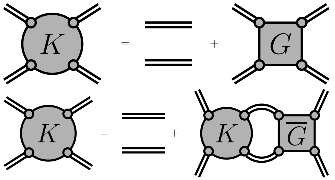
Dyson-Schwinger equation for quark propagator



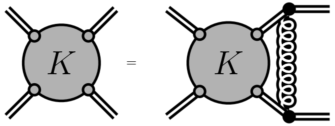
rainbow approximation



Bethe-Salpeter equation



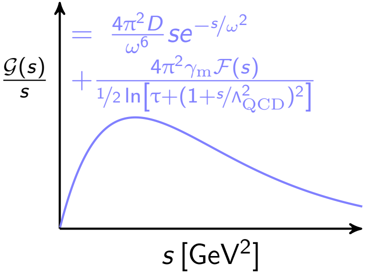
homogeneous, rainbow-ladder aproximation



Model and Strategy

. . . so far no comprehensive attempt at RL meson phenomenology

Maris-Tandy interaction

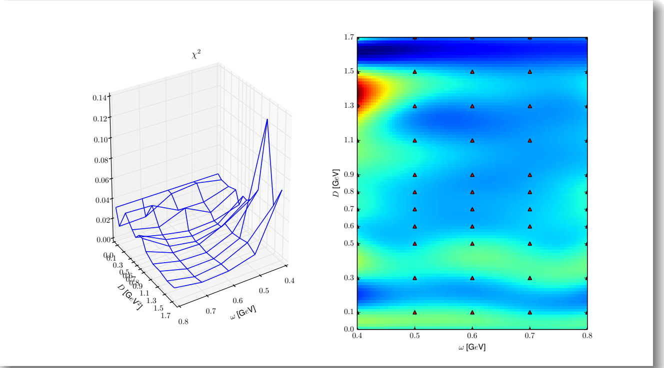


- application to systems where corrections to RL are expected to be least important → bottomonium
- leave functional and UV form unchanged
- allow for more freedom in the effective interaction → quark mass dependence, vary ω and D independently
- include lowest radial excitations
- $J = 0, \dots, 2$

. . . is that good enough?

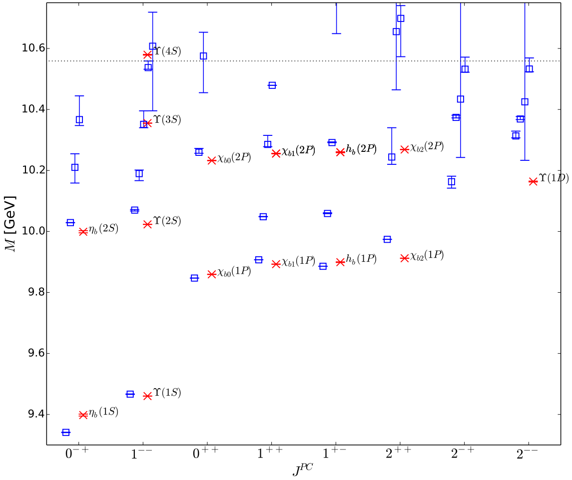
Bottomonium

- evaluate splittings at $(\omega - D)$-grid
- find minimal $\chi^2(\omega, D) = \sum_{\text{splittings}}(\Delta M_{\text{exp}} - \Delta M_{\text{th}})^2$
- find minimal $\bar{\chi}^2(m_q) = \sum_{\text{groundstates}}(M_{\text{exp}} - M_{\text{th}})^2$ for optimal (ω, D)



[C. Popovici, T. Hilger, M. Gómez-Rocha, A. Krassnigg, FBS, arXiv:1407.7970 (2014).]

Bottomonium



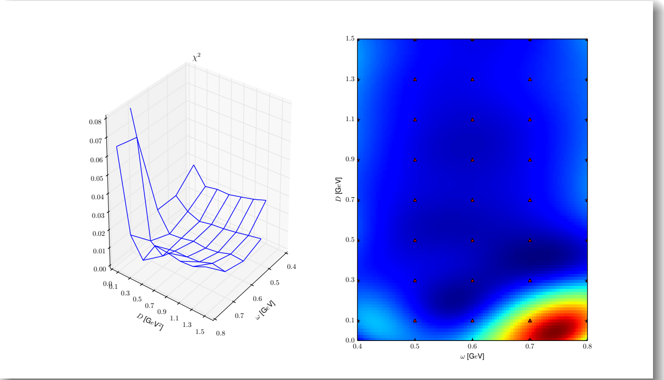
× experiment

- good identification of states
- well reproduced splittings (excitations, level orderings)

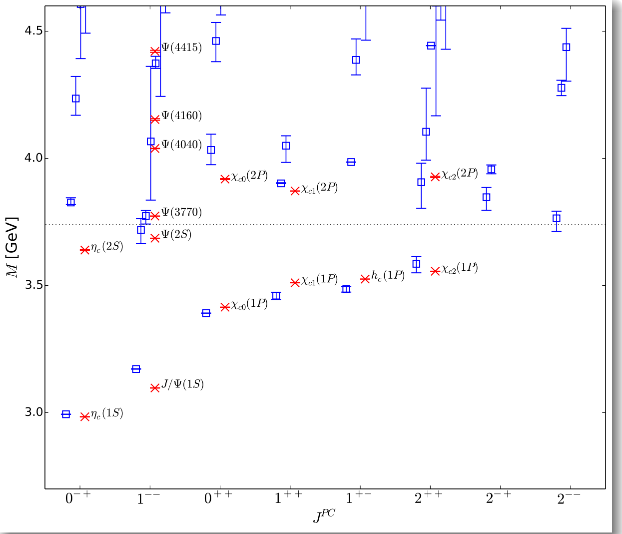
[T. Hilger, C. Popovici, M. Gómez-Rocha, A. Krassnigg, Phys. Rev. D **91**: 034013, 2015.]

- $m_b = 3.635$ GeV at $\mu = 19$ GeV, $\omega = 0.7$ GeV, $D = 1.3$ GeV^2

Charmonium



Charmonium



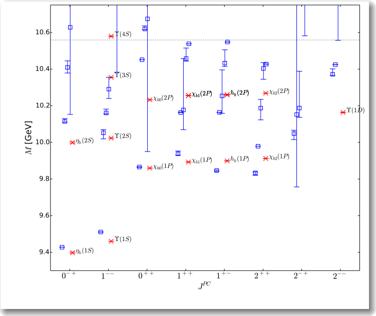
[T. Hilger, C. Popovici, M. Gómez-Rocha, A. Krassnigg, Phys. Rev. D **91**: 034013, 2015.]

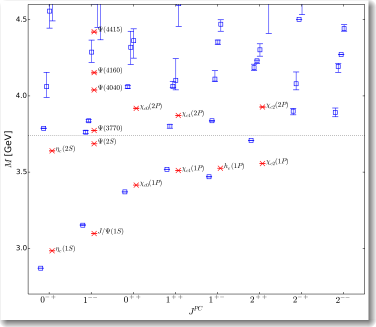
× experiment

- no extra states
- excellently reproduced splittings, in particular 1^{--}

- $m_c = 0.855$ GeV at $\mu = 19$ GeV, $\omega = 0.7$ GeV, $D = 0.5$ GeV^2

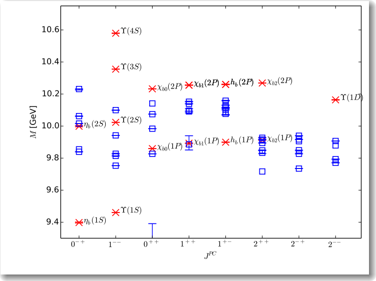
Exchanging parameters

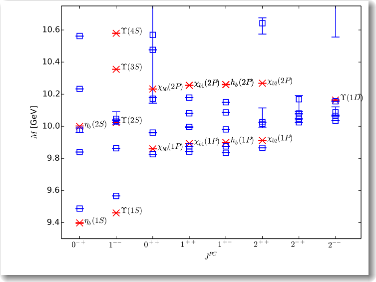


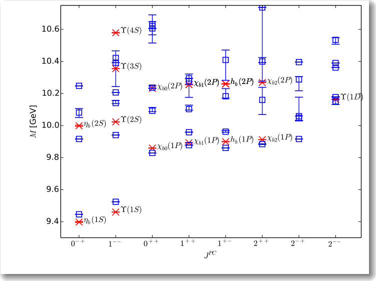


[T. Hilger, C. Popovici, M. Gómez-Rocha, A. Krassnigg, Phys. Rev. D **91**: 034013, 2015.]

Standard Maris-Tandy parameters for bottomonium

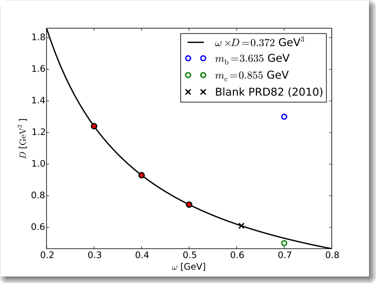


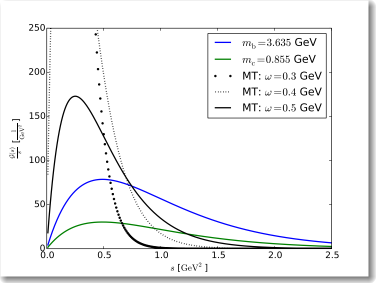




- $\omega \times D = 0.372\mathrm{GeV}^3$
- $\omega \in \{0.3, 0.4, 0.5\}$
- constrained by m_π and $\langle \bar{q}q \rangle$

Interaction

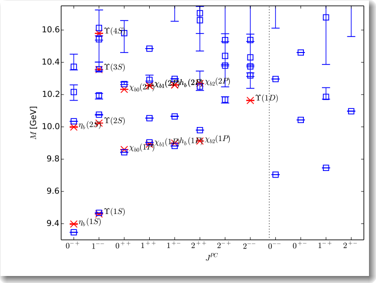


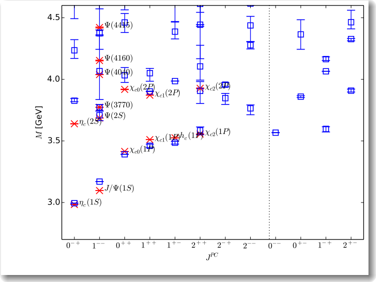


$m_{\rm b} = 3.635$ GeV: $\omega = 0.7$ GeV, $D = 1.3$ GeV^2

$m_{\rm c} = 0.855$ GeV: $\omega = 0.7$ GeV, $D = 0.5$ GeV^2

Exotics: Outlook





- too low compared to quark model predictions, in particular 0^{--}, 1^{-+}
- lower than or $\approx I = 1$ groundstates

Decay constants: Outlook

state	J^{PC}	ground states		radial excitations			
				1st		2nd	
		f calc.	f exp.	f calc.	f exp.	f calc.	f exp.
η_c	0^{-+}	401	338	244(12)	243()	145(145)	–
J/Ψ	1^{--}	450	416	30(3)	296()	118(91)	100()
η_b	0^{-+}	773	–	419(8)	–	534(57)	–
Υ	1^{--}	768	701	467(17)	497()	41(7)	430()

Summary and Outlook

- quark mass dependence of effective interaction
- optimized rainbow-ladder DS-BS study describes ground states and lowest radial excitations
- extra states in vector- and axial-vector channel for bottomonium
- improve state identification (beyond J^{PC} and mass)
- stay tuned: . . . *exotics* . . . decay constants . . . light quarks . . .

[T. Hilger, C. Popovici, M. Gómez-Rocha, A. Krassnigg, Phys. Rev. D **91**: 034013, 2015.]
[C. Popovici, T. Hilger, M. Gómez-Rocha, A. Krassnigg, FBS, arXiv:1407.7970 (2014).]

Four quark condensates and open charm mesons in the medium

together with:
Thomas Buchheim and Burkhard Kämpfer
(Helmholtz-Zentrum Dresden-Rossendorf and TU Dresden)

and
Stefan Leupold (Uppsala University)

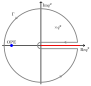
Hadron physics and QCD sum rules

current-current correlator

$$\Pi_{\mu\nu}(q) = i \int d^4x e^{iqx} \langle \mathrm{T} \left[j_\mu(x) \, (j_\nu(0))^\dagger \right] \rangle$$

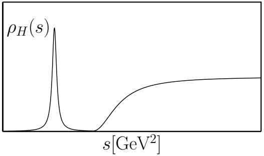
dispersion relation

$$\Pi(q^2) = \frac{1}{\pi} \int_0^\infty \mathrm{d}s \frac{\Delta\Pi(s)}{s-q^2}$$



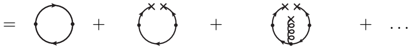
separation of scales

spectral density ↔ hadronic properties



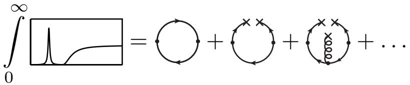
operator product expansion

$$= C_1(q) + C_2(q)\langle \bar{q}q \rangle + C_3(q)\langle \bar{q} g \sigma \mathscr{G} q \rangle + \ldots$$

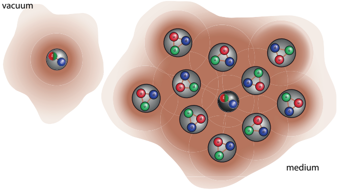


QCD condensates:

- encode medium dependence
- order parameters of chiral symmetry phase transition

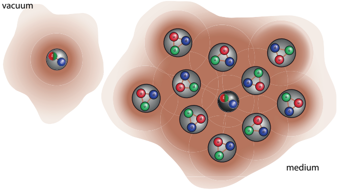


Motivation

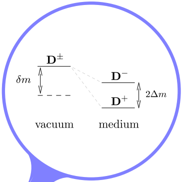


Motivation

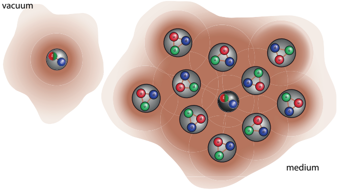
medium
modifications



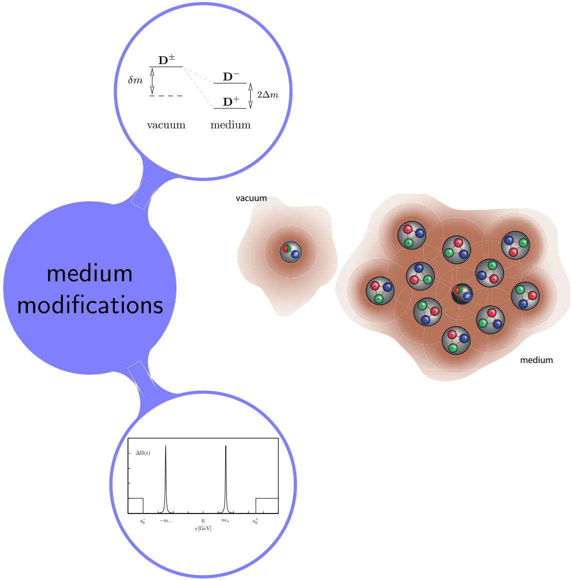
Motivation



medium modifications

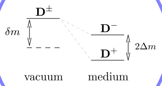


Motivation

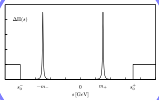


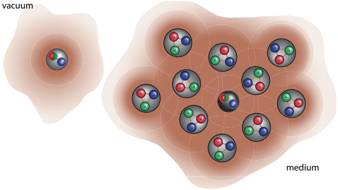
$\mathbf{D}^{\pm}$
δm
$\mathbf{D}^{-}$
$\mathbf{D}^{+}$
$2\Delta m$
vacuum
medium
medium modifications
vacuum
medium
$\Delta\Pi(s)$
s_0^-
$-m_-$
0
m_+
s_0^+
s [GeV]

Motivation



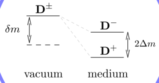
medium modifications



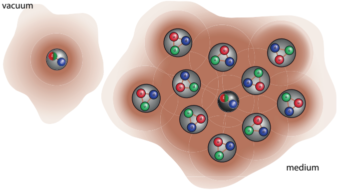


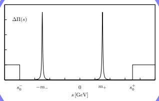
DχSB

Motivation



medium modifications

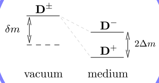




properties of matter under restoration

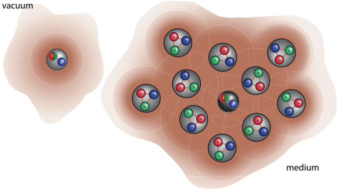
DχSB

Motivation

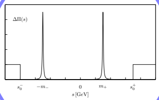


properties of matter under restoration

medium modifications



DχSB

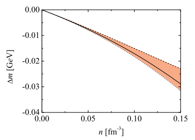


order parameters $\langle \bar{q} q \rangle \;\rightarrow\!\times \;\; \times\!\!\rightarrow$

Results I

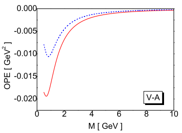
(pseudo-)scalar heavy-light quark mesons at finite density

[TH, R. Thomas, B. Kämpfer, Phys. Rev. C **79** (2009) 025202]
[S. Zschocke, TH, B. Kämpfer, Eur. Phys. J. A **47** (2011) 151]
[TH, B. Kämpfer, Nucl. Phys. B Proc. Suppl. **207-208** (2010) 025202]
[TH, B. Kämpfer, Conf. Proc. Italian Phys. Soc. **99** (2010)]
[B. Kämpfer, TH, H. Schade, R. Schulze, G. Wolf, PoSBormio **2010**]
[R. Rapp et al., *In-medium excitations*, Lect. Notes Phys. **814** 335 (2011)]



chiral sum rules for heavy-light quark spin-0 and -1 mesons

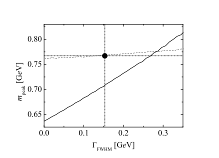
[TH, B. Kämpfer, S. Leupold, Phys. Rev. C **84** (2011) 045202]
[TH, T. Buchheim, B. Kämpfer, S. Leupold, Prog. Part. Nucl. Phys. **67** (2012) 188]
[TH, R. Schulze, B. Kämpfer, J. Phys. G: Nucl. Part. Phys. **37** (2010) 094054]
[TH, B. Kämpfer, Nucl. Phys. Proc. Suppl. **207-208** (2010) 277]



Results II

impact of DχSB order parameters on the ρ meson and implications of chirally symmetric sum rules

[TH, R. Thomas, B. Kämpfer, S. Leupold, Phys. Lett. B **709** (2012) 200]



Current projects

four-quark condensates in heavy-light quark meson QCD sum rules [T. Buchheim, TH, B. Kämpfer, Phys. Rev. C91 (2015)]

- $\rightarrow$ order parameters
- $\rightarrow$ spin-0 and -1
- $\rightarrow$ finite density
- $\rightarrow$ chiral sum rules

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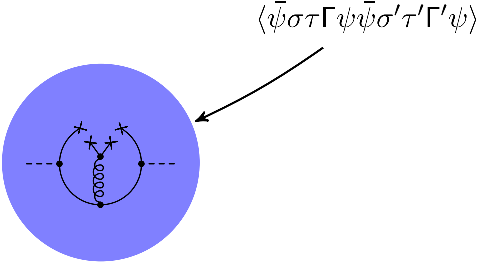

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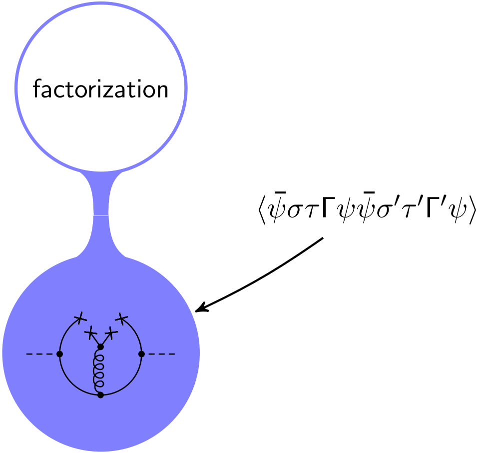

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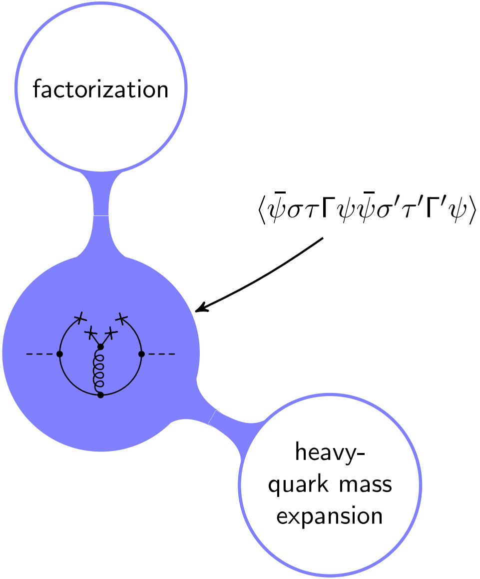

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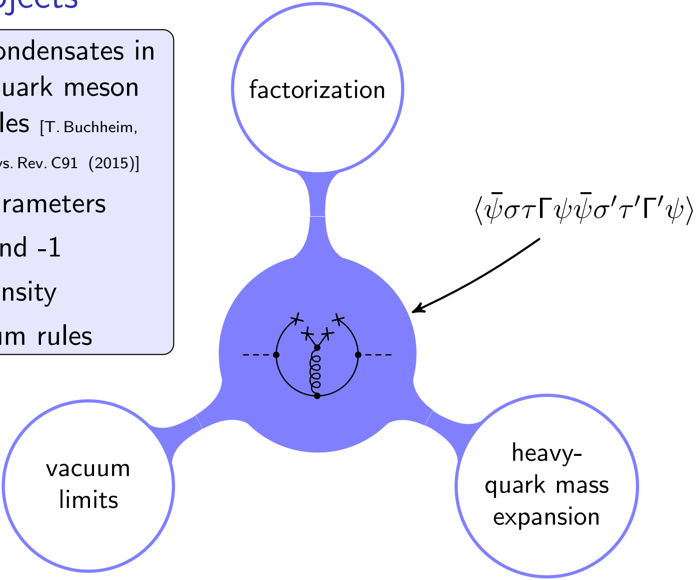

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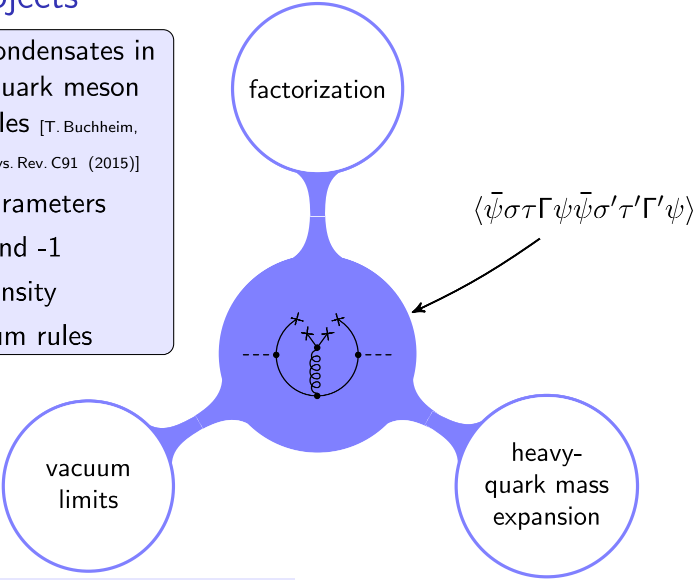

[T. Buchheim, TH, B. Kämpfer, J. Phys. Conf. Ser. 503 (2014)]
[T. Buchheim, TH, B. Kämpfer, E.P.J. WoC 81 (2014)]

[T. Buchheim, TH, B. Kämpfer, Nucl. Phys. Proc. Suppl.]

Summary and Outlook

light four-quark condensates for D mesons in medium

[T. Buchheim, TH, B. Kämpfer, Phys. Rev. C91 (2015)]

- heavy-quark mass expansion of four-quark condensates
- continuous transition from medium to vacuum $\rightarrow$ algebraic vacuum limits

[T. Buchheim, TH, B. Kämpfer, J. Phys. Conf. Ser. 503 (2014)]
[T. Buchheim, TH, B. Kämpfer, Nucl. Phys. Proc. Suppl.]
[T. Buchheim, TH, B. Kämpfer, E. P. J. WoC 81 (2014)]