

Der Wissenschaftsfonds.

Exotic and non-exotic quarkonium properties within the Dyson-Schwinger–Bethe-Salpeter equation approach

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together with:

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Krassnigg (Univ. Graz)

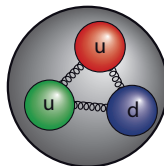
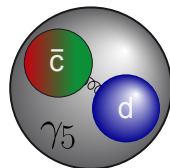
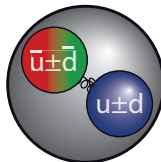
supported by:

Austrian Science Fund (FWF) project no. P25121-N27

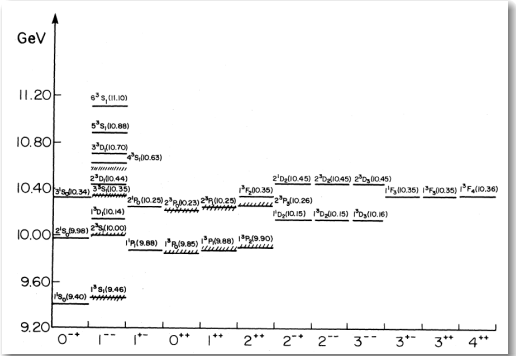
Covariant.ModelsOfHadrons.com

The Problem

- study hadrons as composites of quarks and gluons
 - issues
 - chiral symmetry and $D\chi SB$
 - Poincaré covariance
 - confinement
 - perturbative limit
- calculate observables
- comprehensive phenomenology



Motivation



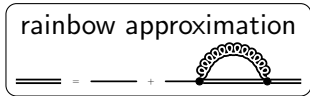
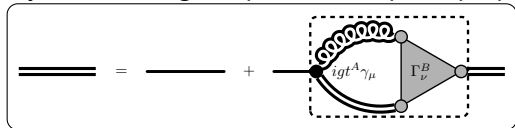
[Bottomonium by Godfrey, Isgur, 1985]

The Tool

- Dyson-Schwinger equations can be used to solve QCD
- Bethe-Salpeter and Faddeev-type equations allow covariant and symmetry-preserving study of bound-state problems
- Infinite set of coupled (and nonlinear) integral equations
- Numerical studies: Truncation \leftrightarrow numerical effort
- Make the truncation respect symmetries
- Construct sophisticated models
- Perform reliable calculations of hadron properties

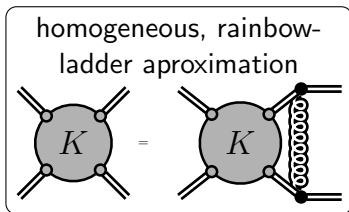
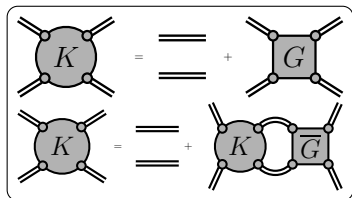
Dyson-Schwinger and Bethe-Salpeter Equations

Dyson-Schwinger equation for quark propagator



$$S^{-1}(p) = S^{(0)-1} + \int dq G_{\mu\nu}(p-q) \gamma_\mu S(q) \gamma_\nu$$

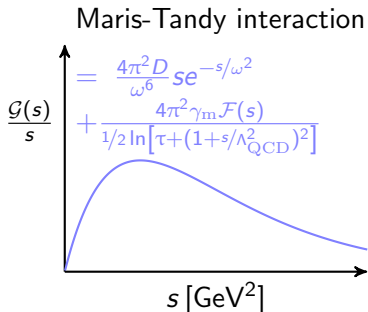
Bethe-Salpeter equation



$$\Gamma(p; P) = \Gamma^{(0)}(p) - \int dq G_{\mu\nu}(p-q) \gamma_\mu S(q_+) \Gamma(q; P) S(q_-) \gamma_\nu$$

Model and Strategy

... so far no comprehensive attempt at RL meson phenomenology

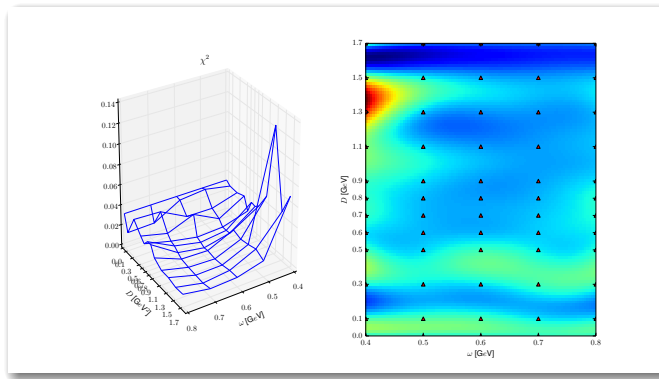


- application to systems where corrections to RL are expected to be least important → bottomonium
- leave functional and UV form unchanged
- allow for more freedom in the effective interaction → quark mass dependence, vary ω and D independently
- include lowest radial excitations
- $J = 0, \dots, 2$

... is that good enough?

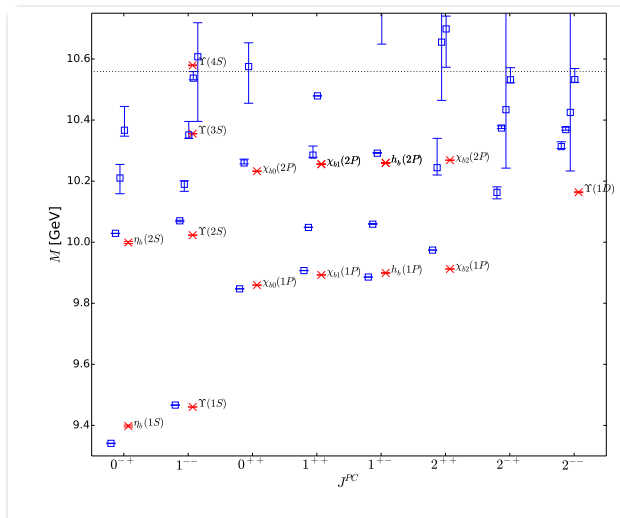
Bottomonium

- evaluate splittings at $(\omega - D)$ -grid
- find minimal $\chi^2(\omega, D) = \sum_{\text{splittings}} (\Delta M_{\text{exp}} - \Delta M_{\text{th}})^2$
- find minimal $\bar{\chi}^2(m_q) = \sum_{\text{groundstates}} (M_{\text{exp}} - M_{\text{th}})^2$ for optimal (ω, D)



[C. Popovici, T. Hilger, M. Gómez-Rocha, A. Krassnigg, FBS, arXiv:1407.7970 (2014).]

Bottomonium



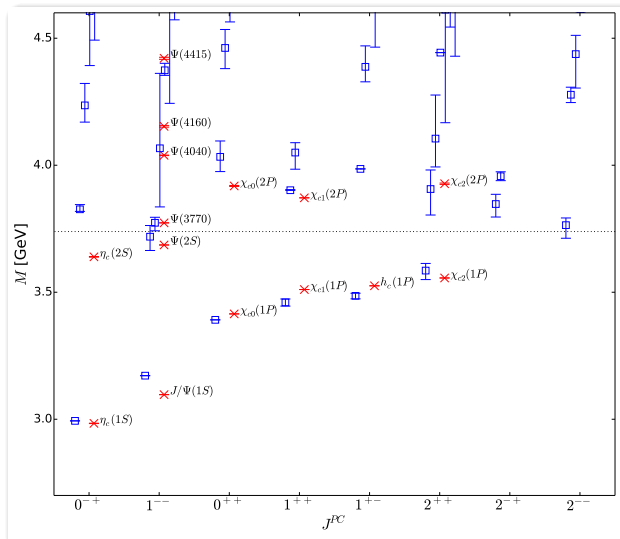
× experiment

- good identification of states
- well reproduced splittings (excitations, level orderings)

[T. Hilger, C. Popovici, M. Gómez-Rocha, A. Krassnigg, Phys. Rev. D **91**: 034013, 2015.]

● $m_b = 3.635$ GeV at $\mu = 19$ GeV, $\omega = 0.7$ GeV, $D = 1.3$ GeV²

Charmonium



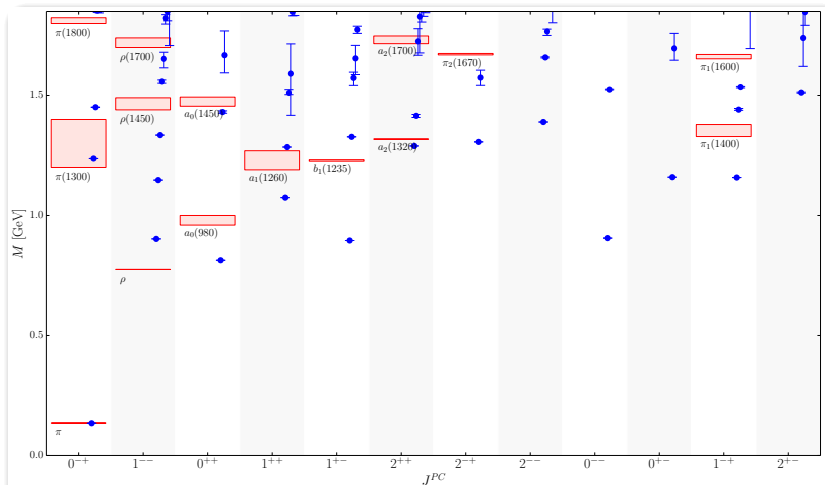
× experiment

- no extra states
- excellently reproduced splittings, in particular 1^{--}

[T. Hilger, C. Popovici, M. Gómez-Rocha, A. Krassnigg, Phys. Rev. D **91**: 034013, 2015.]

● $m_c = 0.855$ GeV at $\mu = 19$ GeV, $\omega = 0.7$ GeV, $D = 0.5$ GeV²

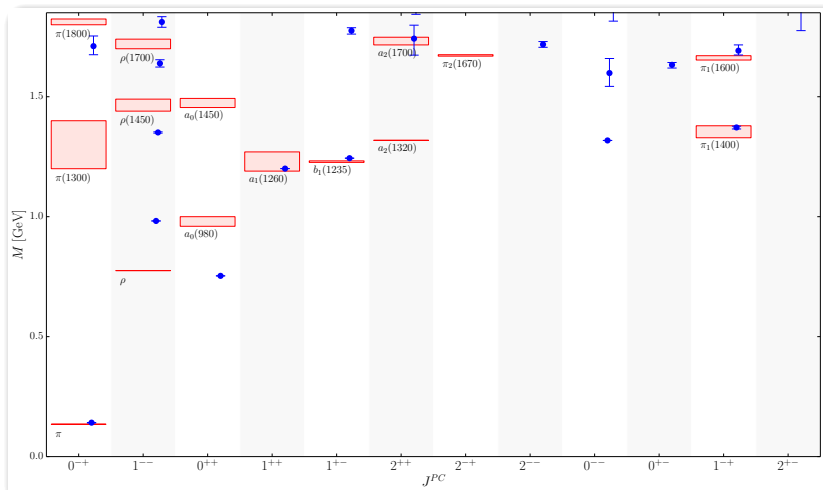
Light Isovector Quarkonium



[T. Hilger, M. Gómez-Rocha, A. Krassnigg, arXiv:1508.07183]

● $m_q = 0.003$ GeV at $\mu = 19$ GeV, $\omega = 0.4$ GeV, $D = 1.7$ GeV²

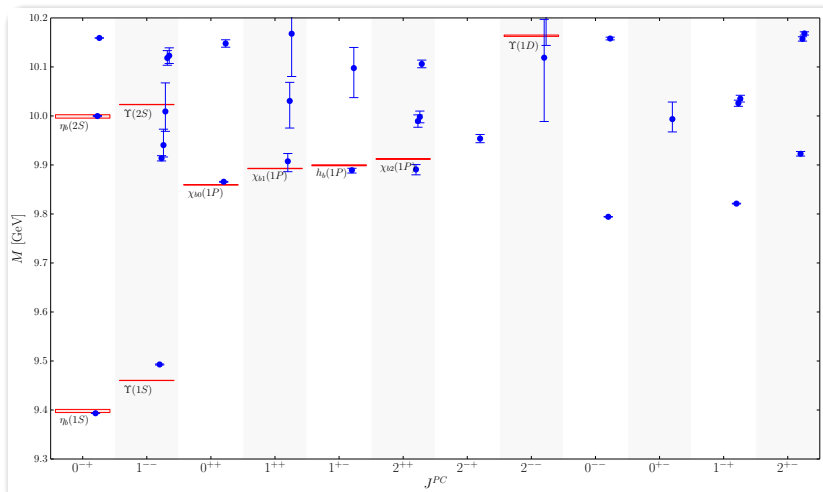
Exotics: light Isovector Quarkonium



[T. Hilger, M. Gómez-Rocha, A. Krassnigg, Phys. Rev. D **91**: 114004, 2015.]

● $m_q = 0.003$ GeV at $\mu = 19$ GeV, $\omega = 0.7$ GeV, $D = 1.4$ GeV²

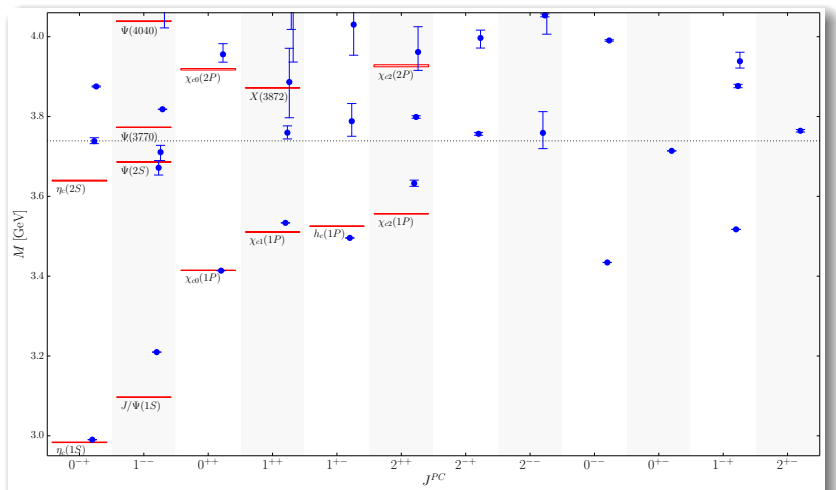
Exotics: Bottomonium



[T. Hilger, C. Popovici, M. Gómez-Rocha, A. Krassnigg, Phys. Rev. D **91**: 114004, 2015.]

● $m_b = 3.635$ GeV at $\mu = 19$ GeV, $\omega = 0.7$ GeV, $D = 0.8$ GeV²

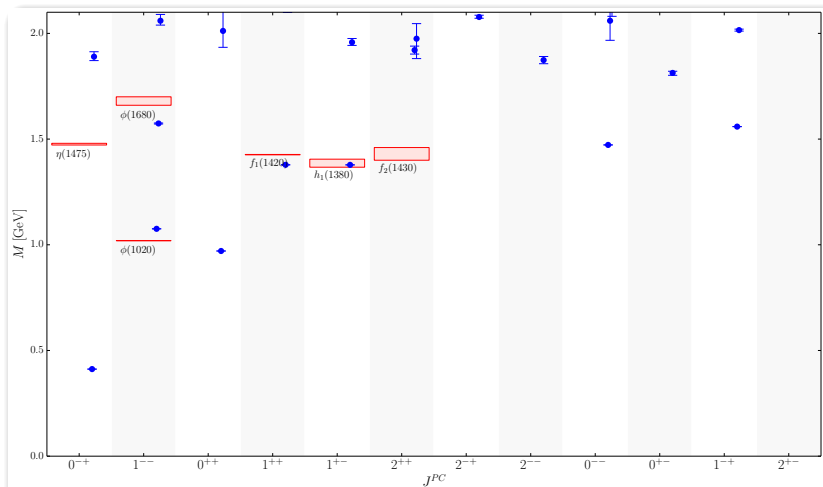
Exotics: Charmonium



[T. Hilger, C. Popovici, M. Gómez-Rocha, A. Krassnigg, Phys. Rev. D **91**: 114004, 2015.]

● $m_c = 0.855 \text{ GeV}$ at $\mu = 19 \text{ GeV}$, $\omega = 0.6 \text{ GeV}$, $D = 0.9 \text{ GeV}^2$

Exotics: Strangeonium



[T. Hilger, M. Gómez-Rocha, A. Krassnigg, arXiv:1508.07183]

● $m_q = 0.070$ GeV at $\mu = 19$ GeV, $\omega = 0.8$ GeV, $D = 1.7$ GeV²

Summary and Outlook

- quark mass dependence of effective interaction
- optimized rainbow-ladder DS-BS study describes ground states and lowest radial excitations
- extra states in vector- and axial-vector channel for bottomonium
- *exotic* charmonium, bottomonium (and light isovector) spectrum
- improve state identification (beyond J^{PC} and mass)
- stay tuned: ... decay constants ... light quarks ... heavy-light quark mesons ...

[T. Hilger, M. Gómez-Rocha, A. Krassnigg, arXiv:1508.07183]

[T. Hilger, C. Popovici, M. Gómez-Rocha, A. Krassnigg, Phys. Rev. D **91**: 114004, 2015.]

[T. Hilger, C. Popovici, M. Gómez-Rocha, A. Krassnigg, Phys. Rev. D **91**: 034013, 2015.]

[C. Popovici, T. Hilger, M. Gómez-Rocha, A. Krassnigg, FBS, arXiv:1407.7970 (2014).]

Four quark condensates and open charm mesons in the medium

together with:

Thomas Buchheim and Burkhard Kämpfer
(Helmholtz-Zentrum Dresden-Rossendorf and TU Dresden)

and

Stefan Leupold (Uppsala University)

Hadron physics and QCD sum rules

current-current correlator

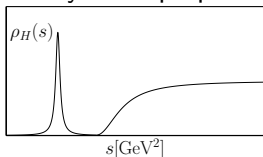
$$\Pi_{\mu\nu}(q) = i \int d^4x e^{iqx} \langle T [j_\mu(x) (j_\nu(0))^\dagger] \rangle$$

dispersion relation

$$\Pi(q^2) = \frac{1}{\pi} \int_0^\infty ds \frac{\Delta\Pi(s)}{s-q^2}$$



spectral density \leftrightarrow hadronic properties



separation of scales

operator product expansion

$$= C_1(q) + C_2(q) \langle \bar{q}q \rangle + C_3(q) \langle \bar{q}g\sigma\mathcal{G}q \rangle + \dots$$

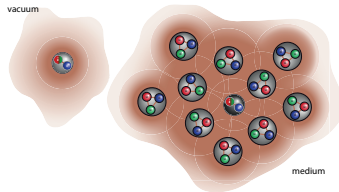
$$= \text{[circle with arrow]} + \text{[circle with arrow and cross]} + \text{[circle with arrow and cross and X]} + \dots$$

QCD condensates:

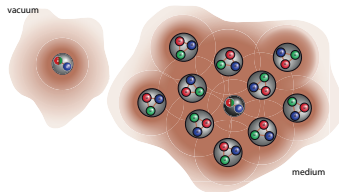
- encode medium dependence
- order parameters of chiral symmetry phase transition

$$\int_0^\infty ds \rho_H(s) = \text{[circle with arrow]} + \text{[circle with arrow and cross]} + \text{[circle with arrow and cross and X]} + \dots$$

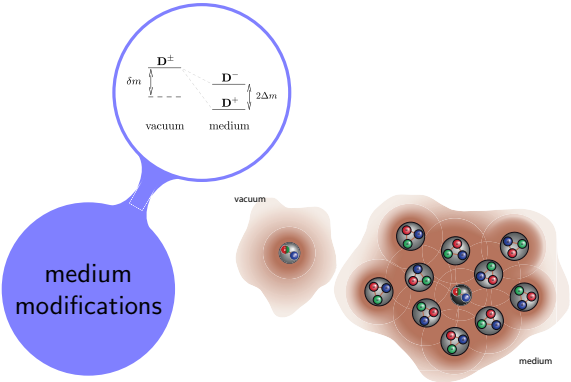
Motivation



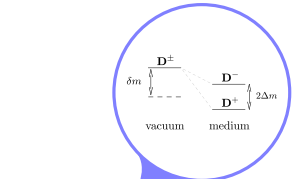
Motivation



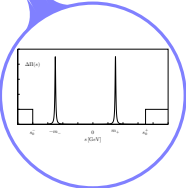
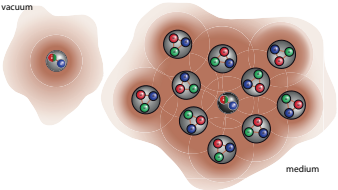
Motivation



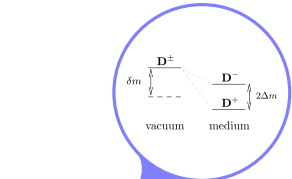
Motivation



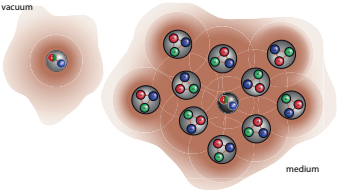
medium modifications



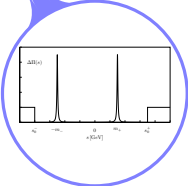
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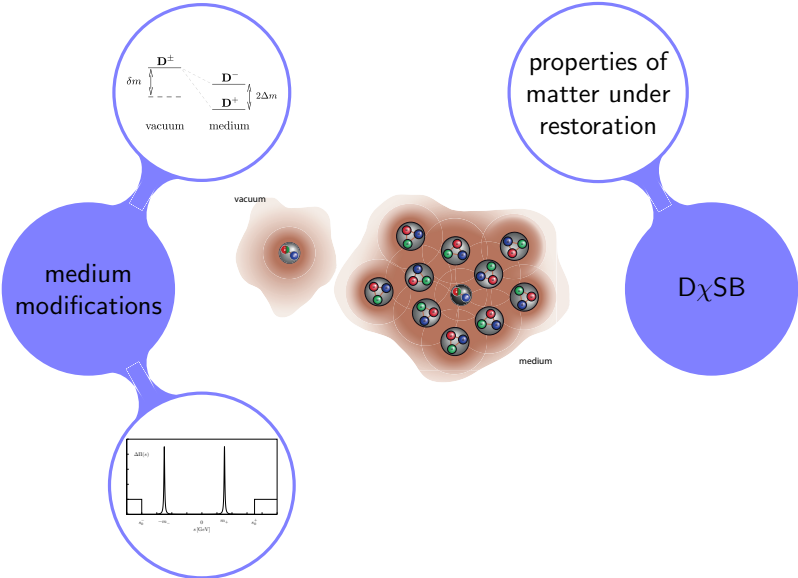
medium modifications



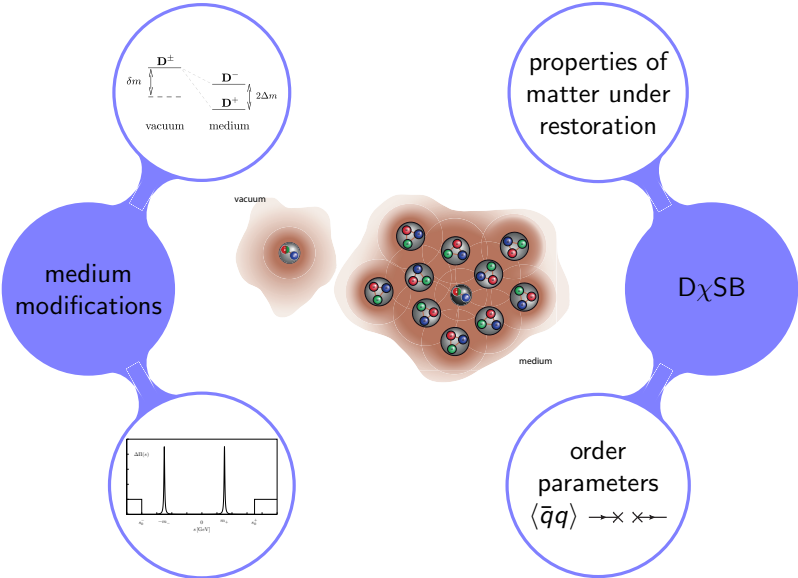
$D\chi SB$



Motivation



Motivation



(pseudo-)scalar heavy-light quark mesons at finite density

[TH, R. Thomas, B. Kämpfer, Phys. Rev. C **79** (2009) 025202]

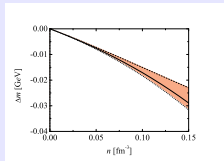
[S. Zschocke, TH, B. Kämpfer, Eur. Phys. J. A **47** (2011) 151]

[TH, B. Kämpfer, Nucl. Phys. B Proc. Suppl. **207-208** (2010) 025202]

[TH, B. Kämpfer, Conf. Proc. Italian Phys. Soc. **99** (2010)]

[B. Kämpfer, TH, H. Schade, R. Schulze, G. Wolf, PoSBormio **2010**]

[R. Rapp et al., *In-medium excitations*, Lect. Notes Phys. **814** 335 (2011)]



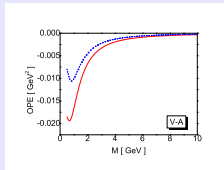
chiral sum rules for heavy-light quark spin-0 and -1 mesons

[TH, B. Kämpfer, S. Leupold, Phys. Rev. C **84** (2011) 045202]

[TH, T. Buchheim, B. Kämpfer, S. Leupold, Prog. Part. Nucl. Phys. **67** (2012) 188]

[TH, R. Schulze, B. Kämpfer, J. Phys. G: Nucl. Part. Phys. **37** (2010) 094054]

[TH, B. Kämpfer, Nucl. Phys. Proc. Suppl. **207-208** (2010) 277]

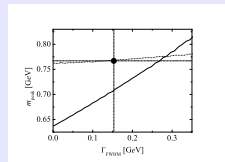


Results II

impact of $D\chi$ SB order parameters on the ρ meson and implications of chirally symmetric sum rules

[TH, R. Thomas, B. Kämpfer, S. Leupold, Phys. Lett. B **709** (2012)

200]



Current projects

four-quark condensates in
heavy-light quark meson

QCD sum rules [T. Buchheim,

TH, B. Kämpfer, Phys. Rev. C91 (2015)]

- order parameters
- spin-0 and -1
- finite density
- chiral sum rules

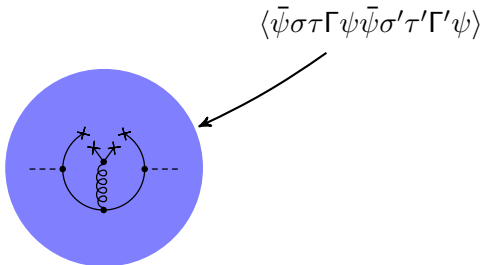
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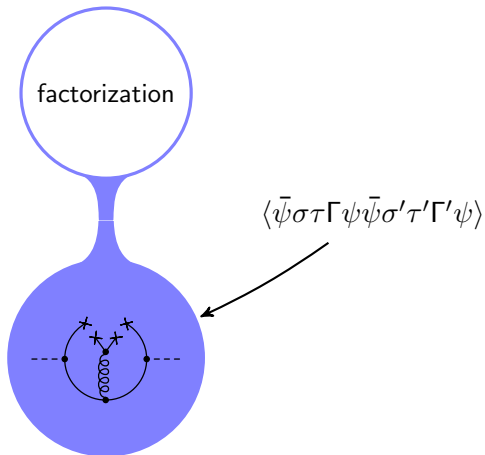
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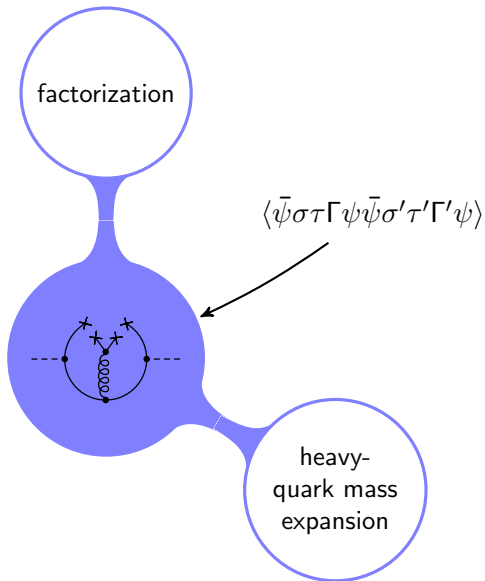
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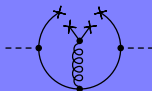
- order parameters
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factorization

$$\langle \bar{\psi} \sigma \tau \Gamma \psi \bar{\psi} \sigma' \tau' \Gamma' \psi \rangle$$

vacuum limits

heavy-quark mass expansion



Current projects

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TH, B. Kämpfer, Phys. Rev. C91 (2015)]

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vacuum limits

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[T. Buchheim, TH, B. Kämpfer, J. Phys. Conf. Ser. 503 (2014)]
[T. Buchheim, TH, B. Kämpfer, E.P.J. WoC81 (2014)]

[T. Buchheim, TH, B. Kämpfer, Nucl. Phys. Proc. Suppl.]

Summary and Outlook

light four-quark condensates for D mesons in medium

[T. Buchheim, TH, B. Kämpfer, Phys. Rev. C91 (2015)]

- heavy-quark mass expansion of four-quark condensates
- continuous transition from medium to vacuum \rightarrow algebraic vacuum limits

[T. Buchheim, TH, B. Kämpfer, J. Phys. Conf. Ser. 503 (2014)]

[T. Buchheim, TH, B. Kämpfer, Nucl. Phys. Proc. Suppl.]

[T. Buchheim, TH, B. Kämpfer, E. P. J. WoC 81 (2014)]